Isochrones fitting:
model uncertainties and limitations

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- Stellar evolution theory is one of the most successful and solid branches of astrophysics

- Fundamental information about stellar populations are inferred by comparing observations with theoretical stellar models

- The accuracy and precision of the parameters inferred by means of any fitting technique depend on the reliability degree of the adopted stellar models

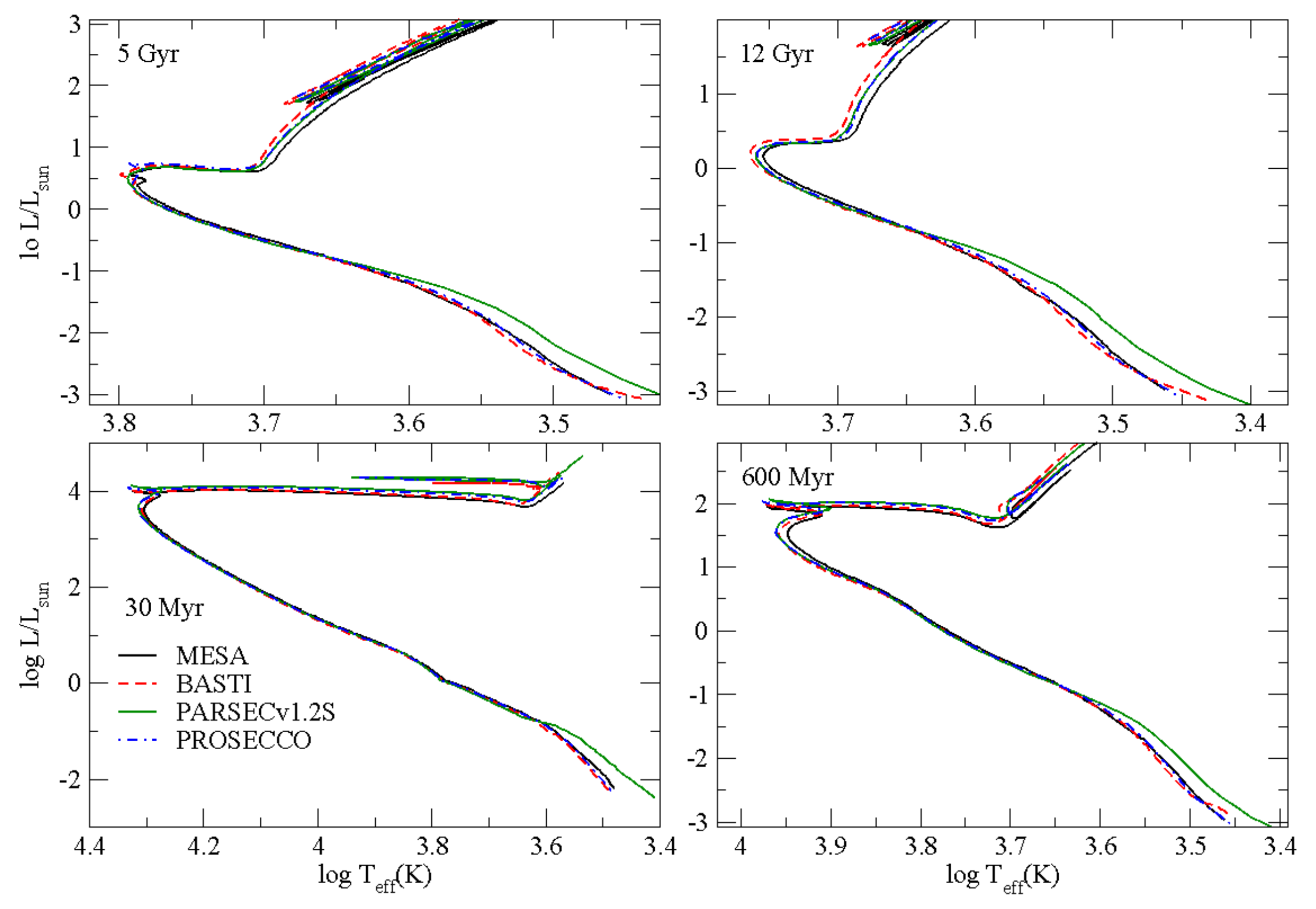
Stellar models depend on…

- input physics (EOS, radiative and conductive opacity, nuclear reaction cross sections, neutrino emission rates, etc.)

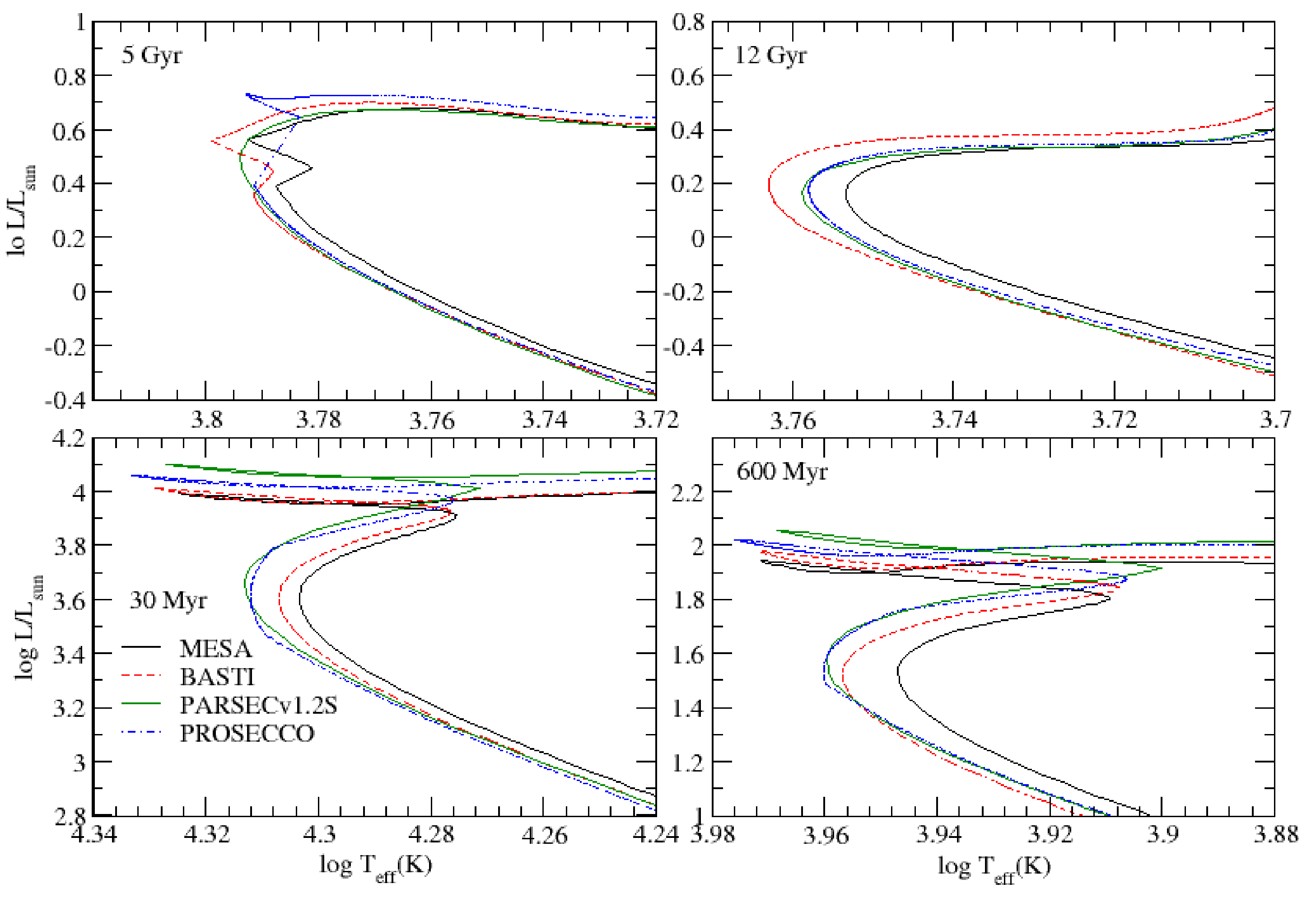
- Macroscopic processes (super-adiabatic convection, overshooting, diffusion, etc.)

- initial chemical composition (Y, Z, elements mixture)

Comparing different models



Comparing different models



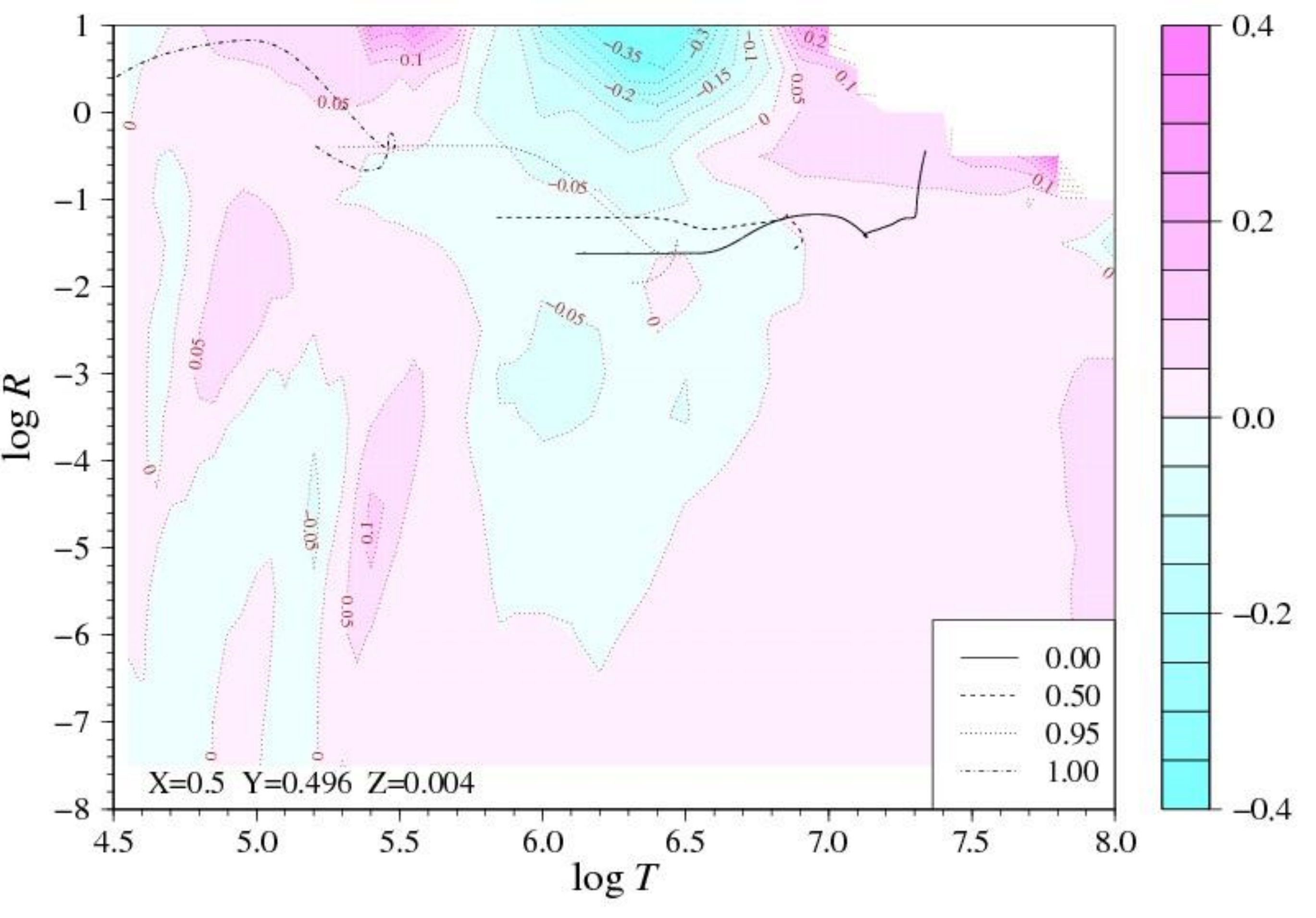
Radiative opacity

- Radiative energy transfer

- Continuously growing accuracy

- $T > 10^4$ K: OPAL *(Iglesias & Rogers 1995)*; Opacity Project *(Badnell et al. 2005)*; OPAS *(Blancard et al. 2012)*; Los Alamos *(Colgan et al. 2016)*
- $T < 10^4$ k: Wichita group *(Ferguson et al. 2005)*

OPAL-OP radiative opacities

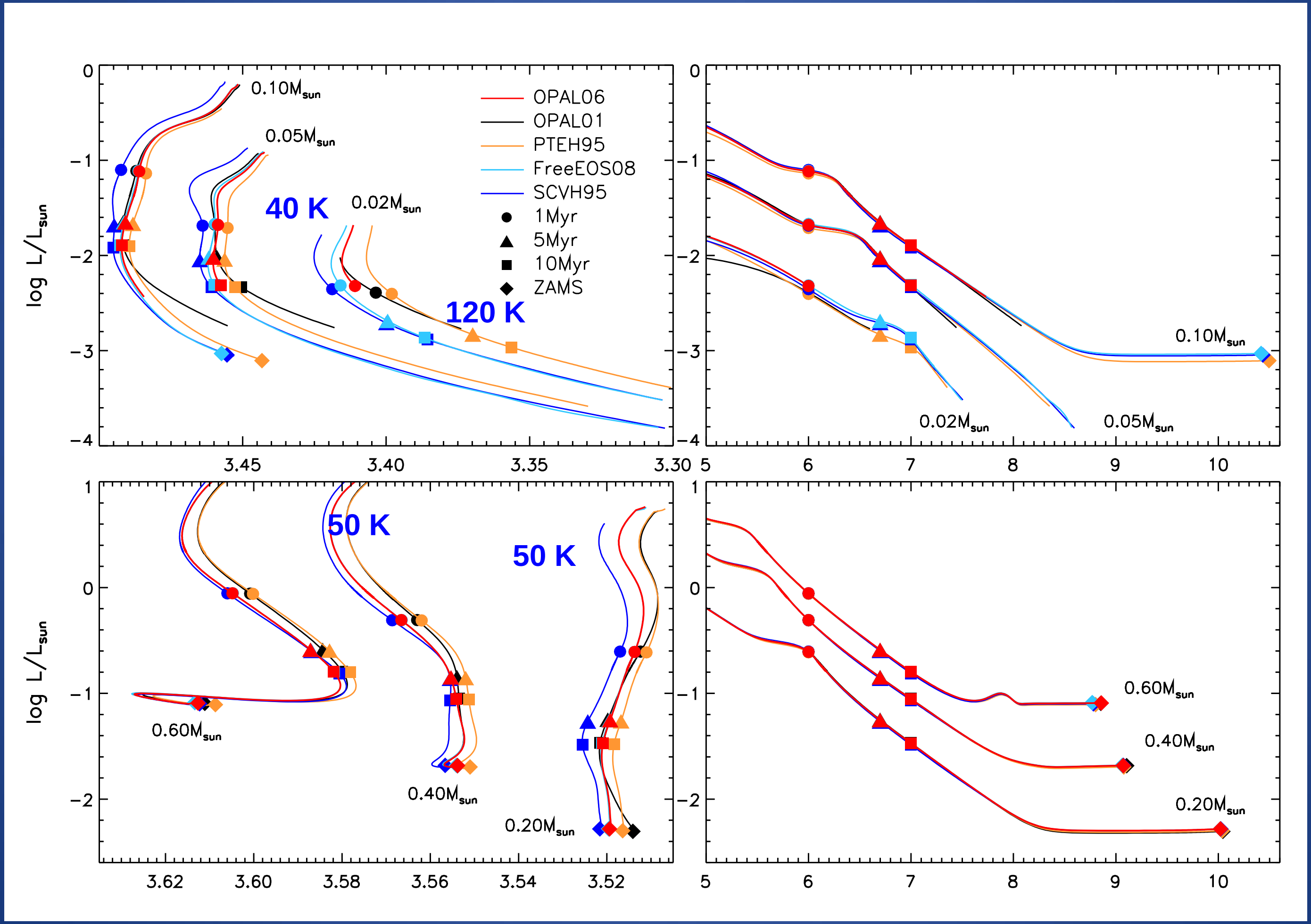


Valle et al. 2013 (see also Badnell et al. 2005)

Equation of state

- $P=P(T, \rho)$; adiabatic gradient, c_p
- OPAL (Rogers & Nayfonov 2002); FreeEOS (Irwin 2008); Saumon et al. 1995

Different EOSs



PISA models

$^{14}N(p,\gamma)^{15}O$

- The slowest process in the CNO cycle

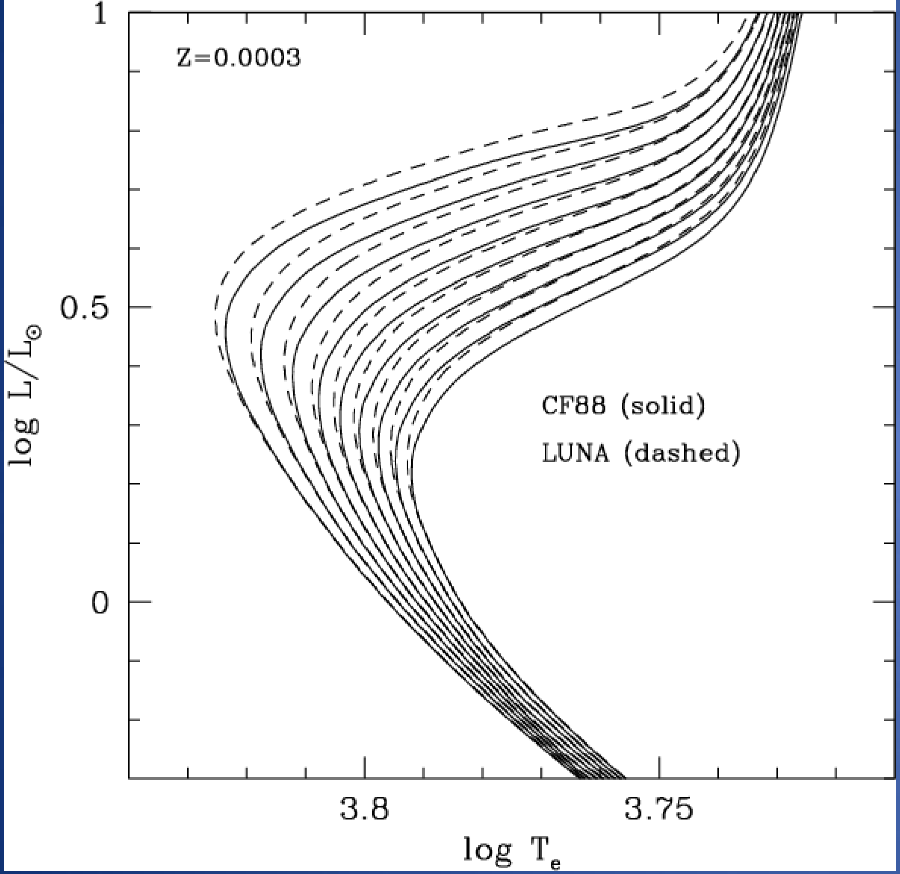
- Age determination of globular clusters *(Degl'Innocenti et al. 2004, Imbriani et al. 2004)*

- Solar neutrino spectrum *(Bahcall & Pinsonneault 2003; Degl'Innocenti et al. 2004; Vinyoles et al. 2017)*

$^{14}N(p,\gamma)^{15}O$

- LUNA collaboration *(Formicola et al. 2003, Imbriani et al. 2005, Marta et al. 2008, 2011)* extended the measurements to the low-energy regime
- The rate is nearly a factor of 2 lower than the NACRE value at low temperatures
- Increase of the age estimate of globular clusters of about 0.7-0.9 Gyr *(Degl'Innocenti et al. 2004; Imbriani et al. 2004; Weiss et al. 2005; Pietrinferni et al. 2010)*

$^{14}N(p,\gamma)^{15}O$



Imbriani et al. 2004

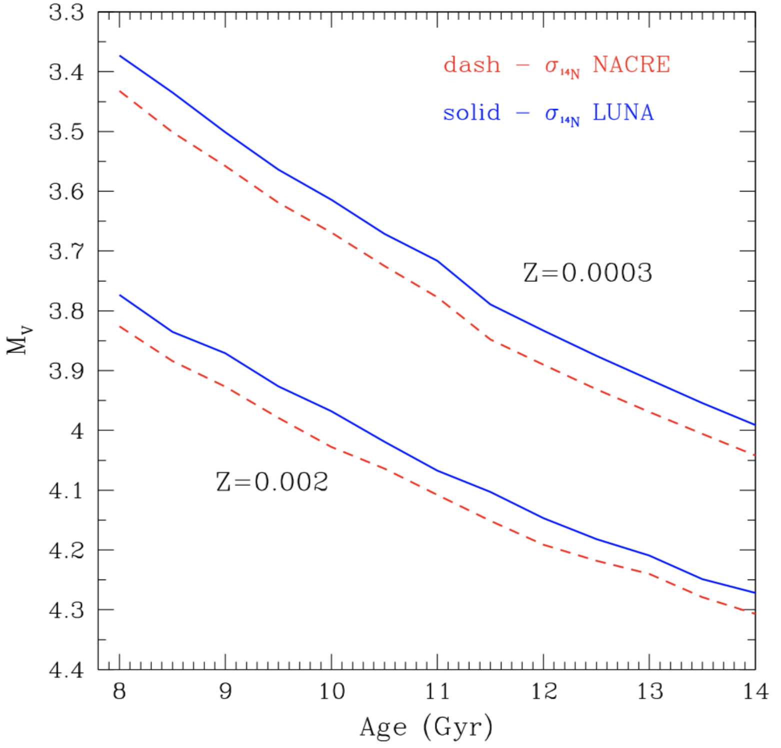


Fig. 2. The absolute V-band magnitude of the MS TO as a function of age for theoretical isochrones with $Z = 0.0003$ and $Z = 0.002$, as derived from stellar models computed with either the LUNA or the NACRE $^{14}N(p, \gamma)^{15}O$ reaction rate.

Pietrinferni et al. 2010

$p(p,e^+\nu)D$

- It drives the efficiency of p-p chain
- Marcucci et al. 2013: updated rate accurate at the level of few per thousand (but see also *Acharya et al. 2016*)
- Tognelli et al. 2015: release for the calculation of the updated p–p rate at the link:

http://astro.df.unipi.it/stellar-models/pprate/

Cumulative uncertainty

$M=0.9\ M_o$ $Z=0.006$ $Y=0.26$ $\alpha_{ml}=1.9$

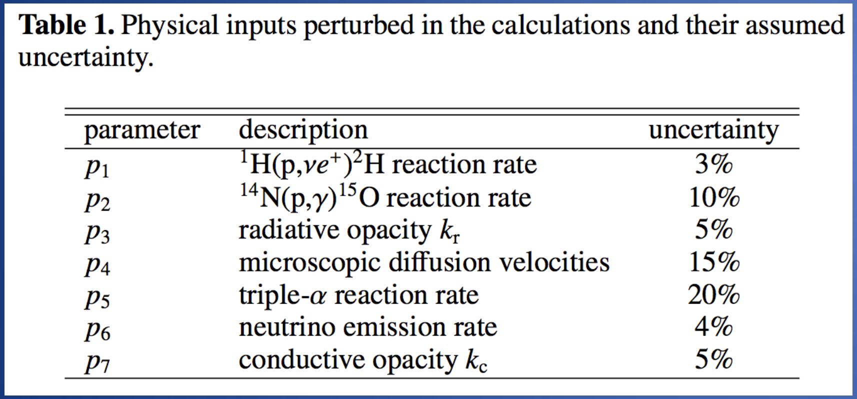


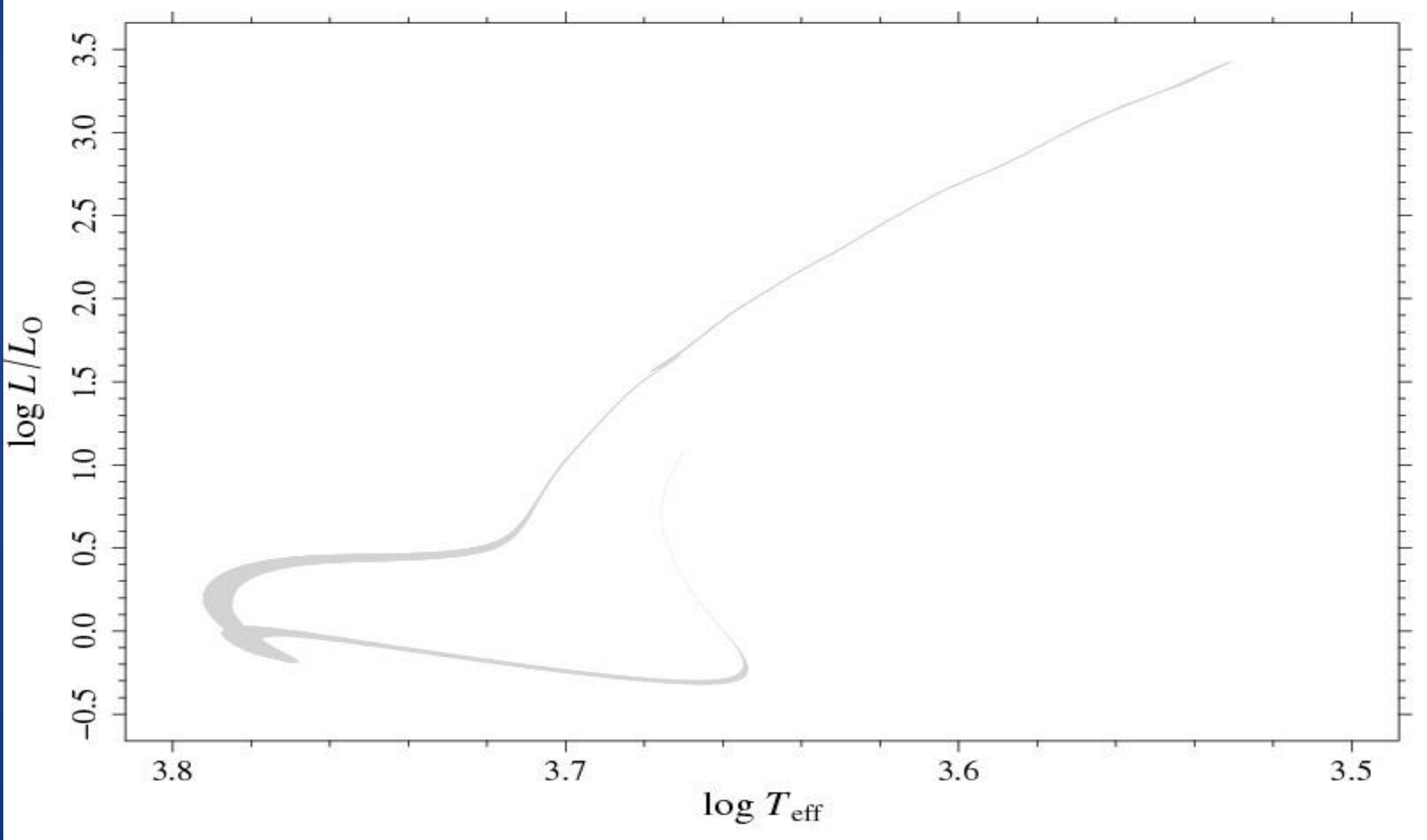
Table 1. Physical inputs perturbed in the calculations and their assumed uncertainty.

parameter	description	uncertainty
p_1	^{1}H(p,νe^+)^{2}H reaction rate	3%
p_2	^{14}N(p,γ)^{15}O reaction rate	10%
p_3	radiative opacity k_r	5%
p_4	microscopic diffusion velocities	15%
p_5	triple-α reaction rate	20%
p_6	neutrino emission rate	4%
p_7	conductive opacity k_c	5%

Valle et al. 2013

- Systematic and symultaneous variation of the main input physics
- Perturbed stellar models (*Chaboyer et al. 1992, 1998, Valle et al. 2013 a,b*)

$M=0.9\ M_o\ Z=0.006\ Y=0.26\ \alpha_{ml}=1.9$



Valle et al. 2013

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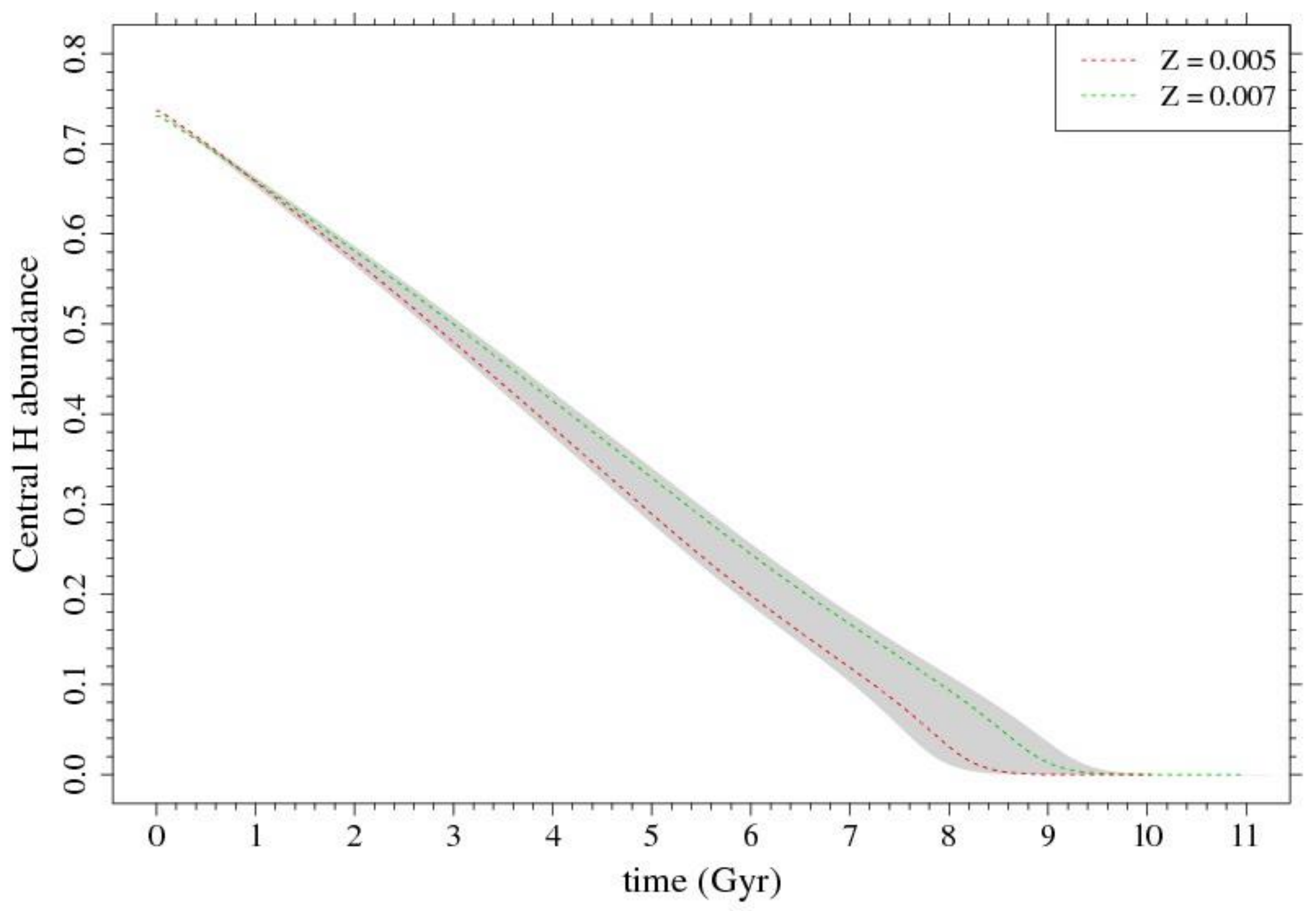


Valle et al. 2013

$M=0.9\ M_o$ $Z=0.006$ $Y=0.26$ $\alpha_{ml}=1.9$

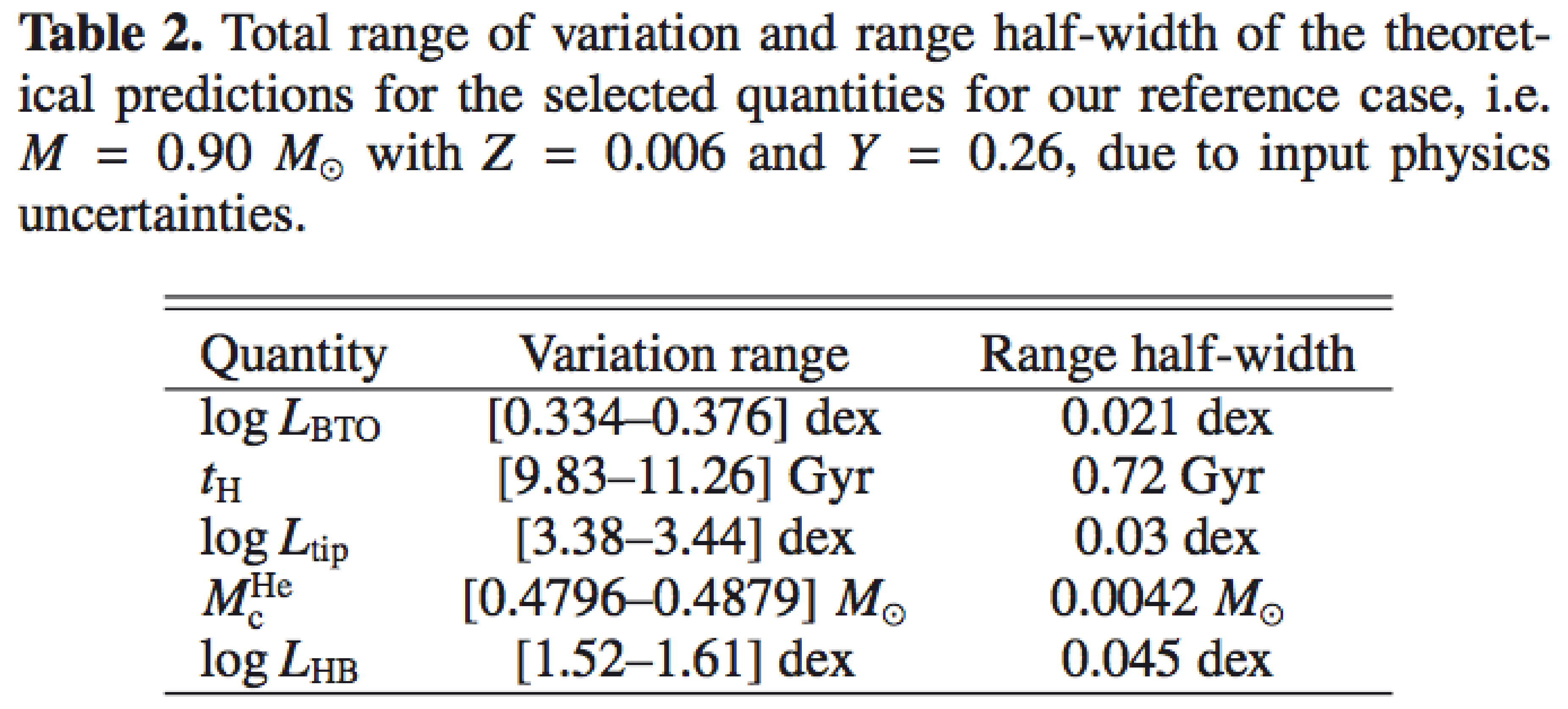
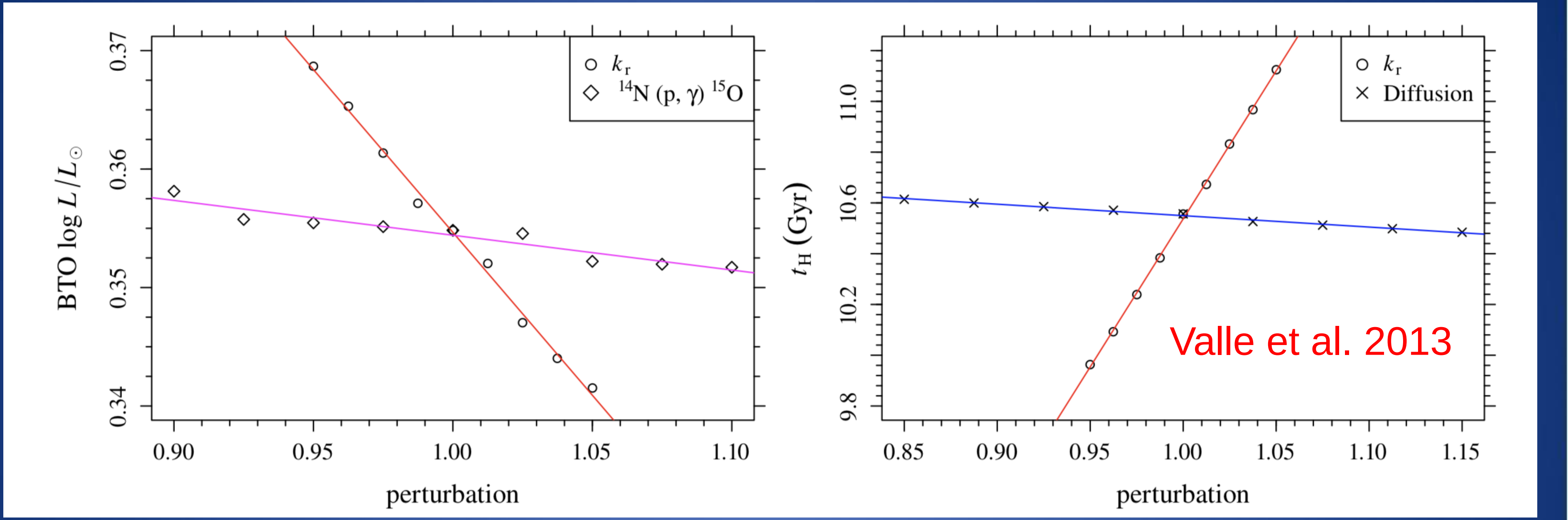


Table 2. Total range of variation and range half-width of the theoretical predictions for the selected quantities for our reference case, i.e. $M = 0.90\ M_\odot$ with $Z = 0.006$ and $Y = 0.26$, due to input physics uncertainties.

Quantity	Variation range	Range half-width
$\log L_{\rm BTO}$	[0.334–0.376] dex	0.021 dex
$t_{\rm H}$	[9.83–11.26] Gyr	0.72 Gyr
$\log L_{\rm tip}$	[3.38–3.44] dex	0.03 dex
$M_{\rm c}^{\rm He}$	[0.4796–0.4879] $M_\odot$	0.0042 $M_\odot$
$\log L_{\rm HB}$	[1.52–1.61] dex	0.045 dex

Valle et al. 2013

$M=0.9\ M_{o}\ Z=0.006\ Y=0.26\ \alpha_{ml}=1.9$



Valle et al. 2013

L_{TO} uncertainty contributions:

- k_r, first

•^{14}N+p, second

σ_{kr} from 5% to 1% to produce the same contribution of the second

t_H uncertainty contributions:

- k_r, first

•Diffusion velocities, second

σ_{kr} from 5% to 0.56% to produce the same contribution of the second

12 Gyr isochrone

Table 8. Range half-width of variation in theoretical predictions of selected quantities for our reference isochrone of 12 Gyr, with $Z = 0.006$ and $Y = 0.26$, due to input physics uncertainties.

Quantity	Range half-width
$\log L_{\mathrm{BTO}}^{\mathrm{iso}}$	0.013 dex
$M_{\mathrm{BTO}}^{\mathrm{iso}}$	0.015 $M_{\odot}$
$\log L_{\mathrm{HB}}/L_{\mathrm{BTO}}^{\mathrm{iso}}$	0.05 dex

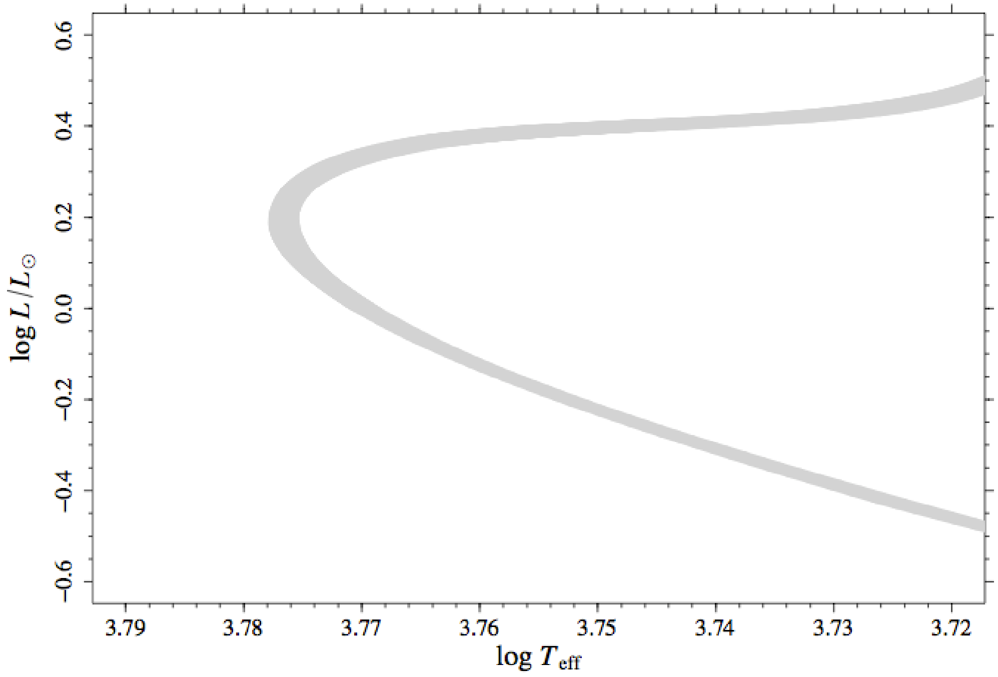
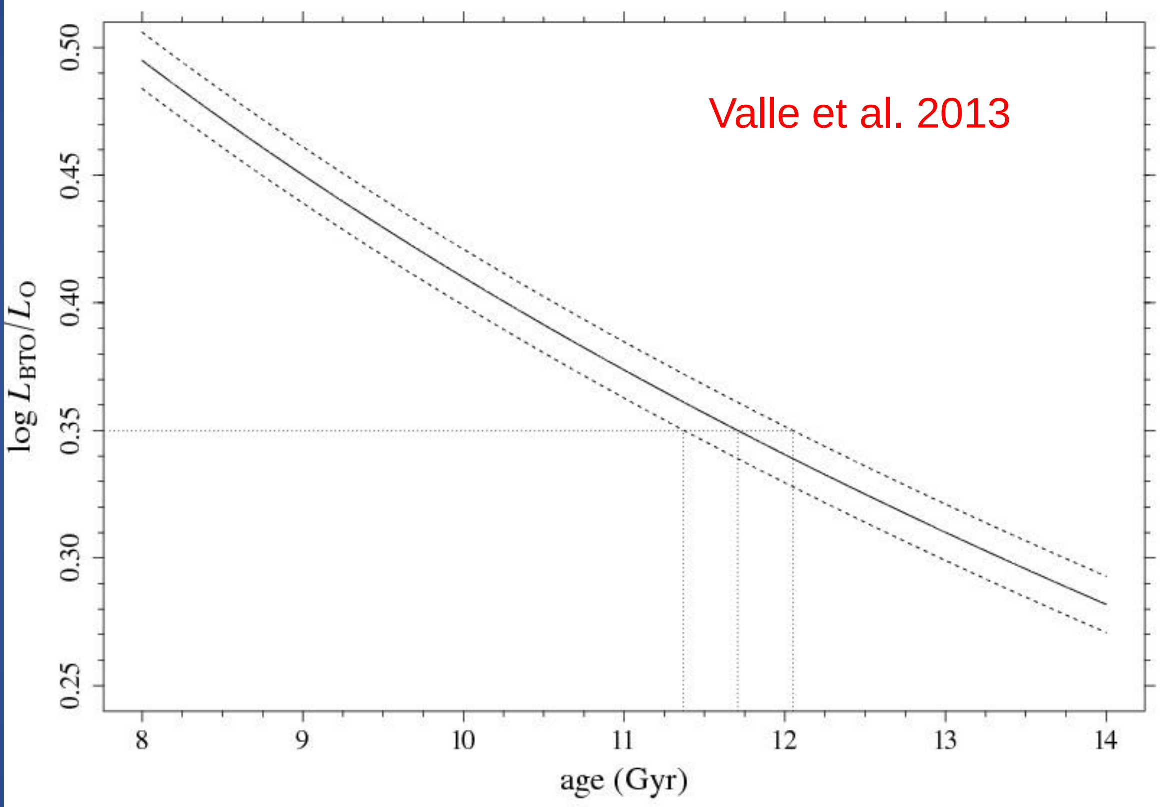


Fig. 10. HR diagram showing the error stripe due to the variation of the physical inputs for a 12.0 Gyr isochrone (zoom of the TO region).

Valle et al. 2013

Isochrones 8-14 Gyr



For a given TO luminosity, the inferred age varies in a range of ≈ +/- 0.375 Gyr

Convection

One of the major and long-standing weaknesses in stellar models

Stellar models are not yet able to accurately predict:

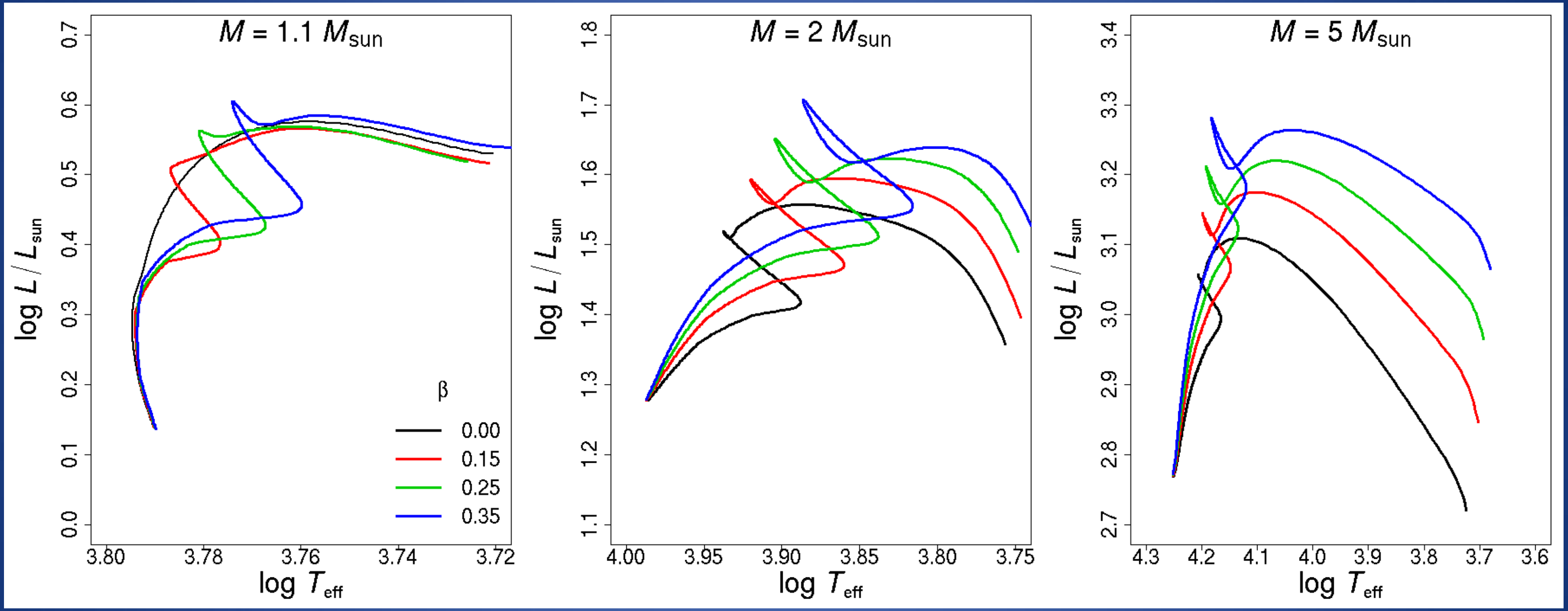
- the extension of convective regions
- the temperature gradient

Core overshooting

Common approach in stellar codes is:

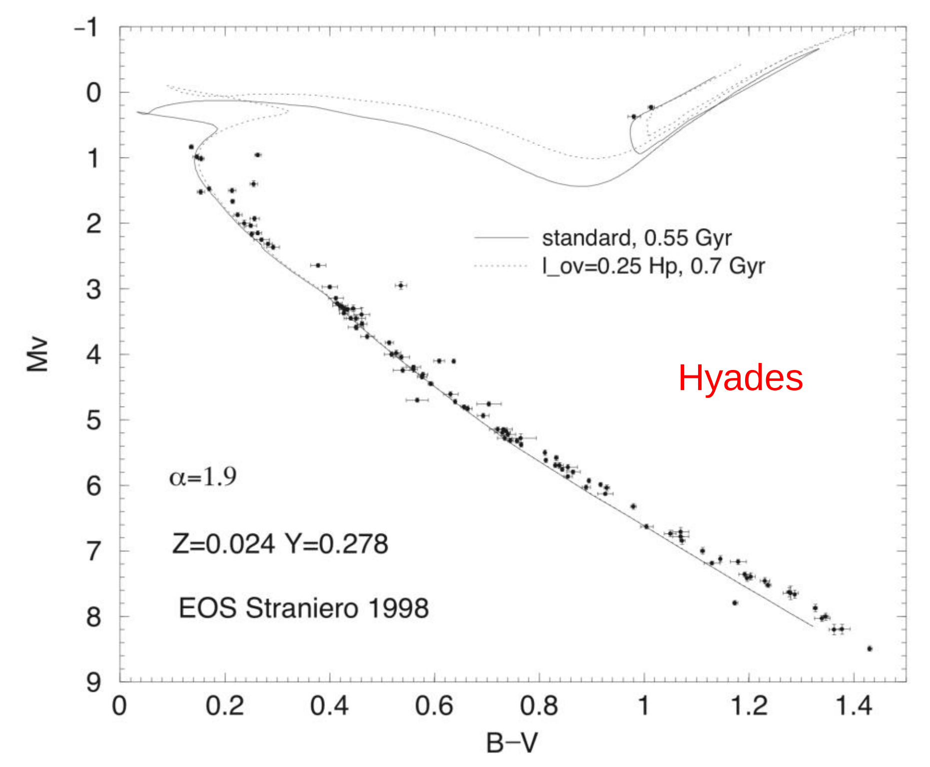
- To determine the Schwarzschild border
- To allow for an overshooting region whose extension depends on a free parameter proportional to the pressure scale height at the Schwarzschild border *(Saslaw & Schwarzschil 1965, Shaviv & Salpeter 1973; Maeder 1975; Renzini 1987; etc.)*

Core overshooting



PISA models

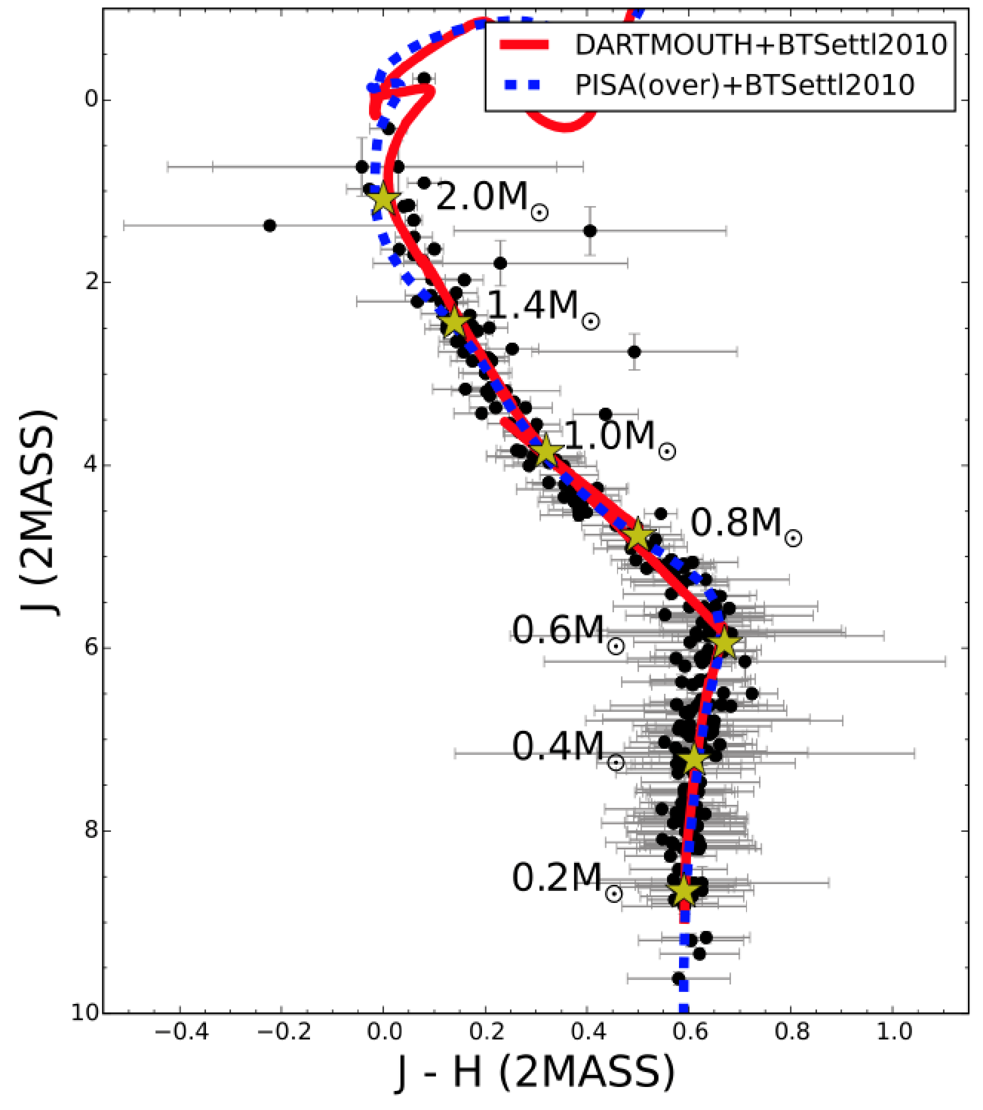
Core overshooting



Castellani, Degl'Innocenti, Prada Moroni 2001

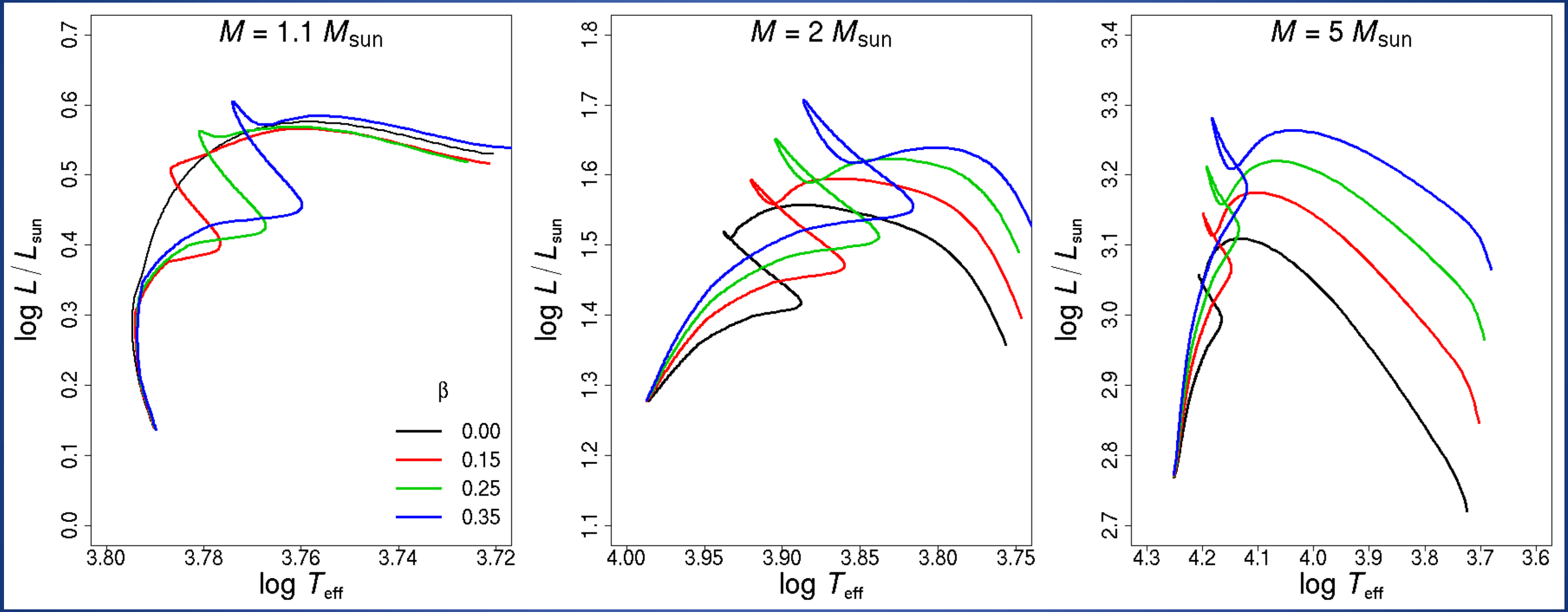
- Affects the age estimate of clusters with TO mass $>1.2\ M_o$
- Increasing the overshooting leads to older ages

Hyades



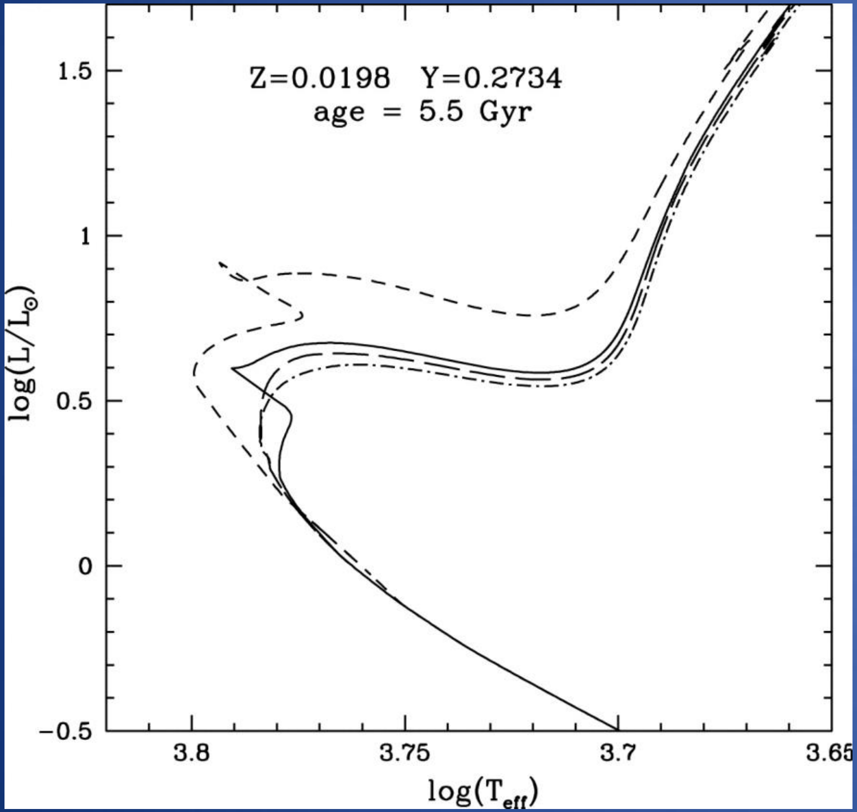
Kopytova et al. 2016

Core overshooting



PISA models

Core overshooting



Pietrinferni et al. 2004

- overshooting prescription for 1.1 M$_o$ < M < 1.5 M$_o$ affects the morphology of isochrones of ≈ 5 Gyr

- Mimicking different ages

Overshooting calibration

- CMD of stellar clusters (Maeder & Mermilliod 1981; Stothers 1991; Schaller et al. 1992;

- Eclipsing binaries *(Andersen et al. 1990; Ribas et al. 2000; Claret 2007; Prada Moroni et al. 2012; Stancliffe et al. 2015; Claret & Torres 2016, 2018; Valle et al. 2017)*

- Asteroseimology *(Deheuvels et al. 2010, 2015; Silva Aguirre et al. 2013)*

Overshooting calibration with eclipsing binaries

Valle et al. 2016, showed that when both members are still in the MS phase:

- errors of 1% in M and 0.5% in R are enough to hamper the overshooting calibration

- The random uncertainty is very large

- The systematic biases suggest caution on the possibility of calibrating overshooting even in the case of a rich sample of binary systems

EB TZ Fornacis

Gallenne et al. 2016 provided very precise mass determination: 0.001 M_o

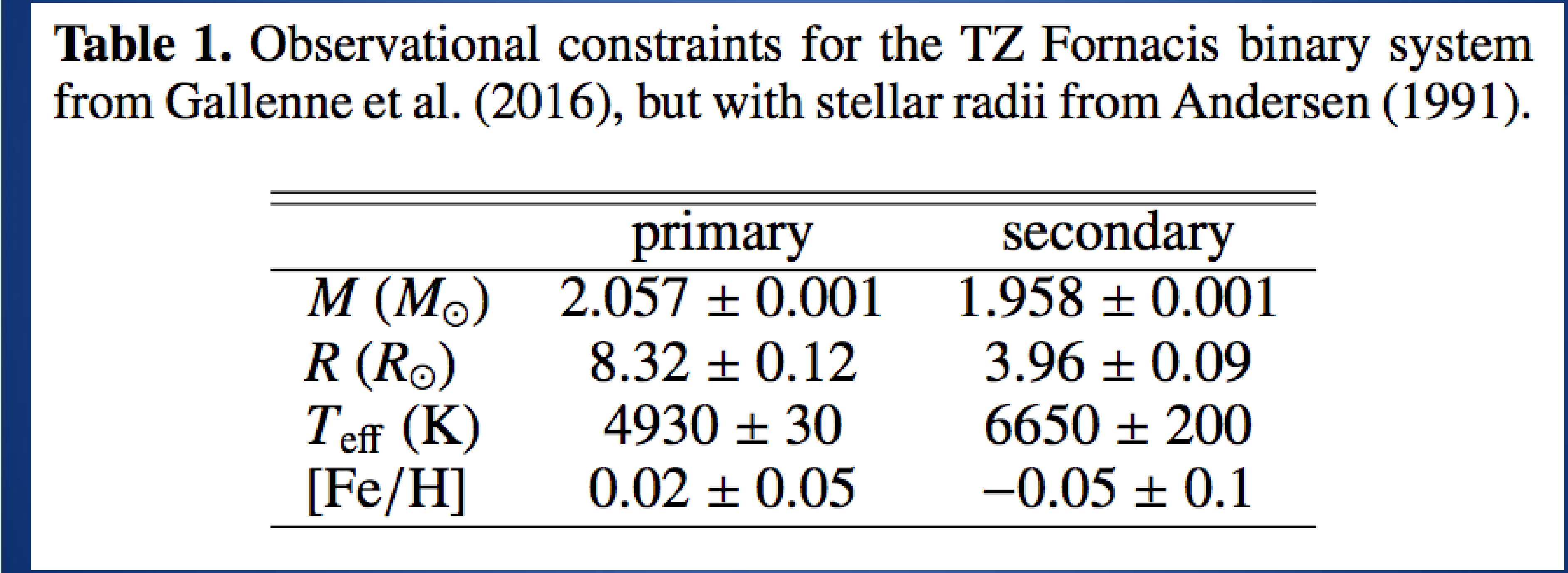
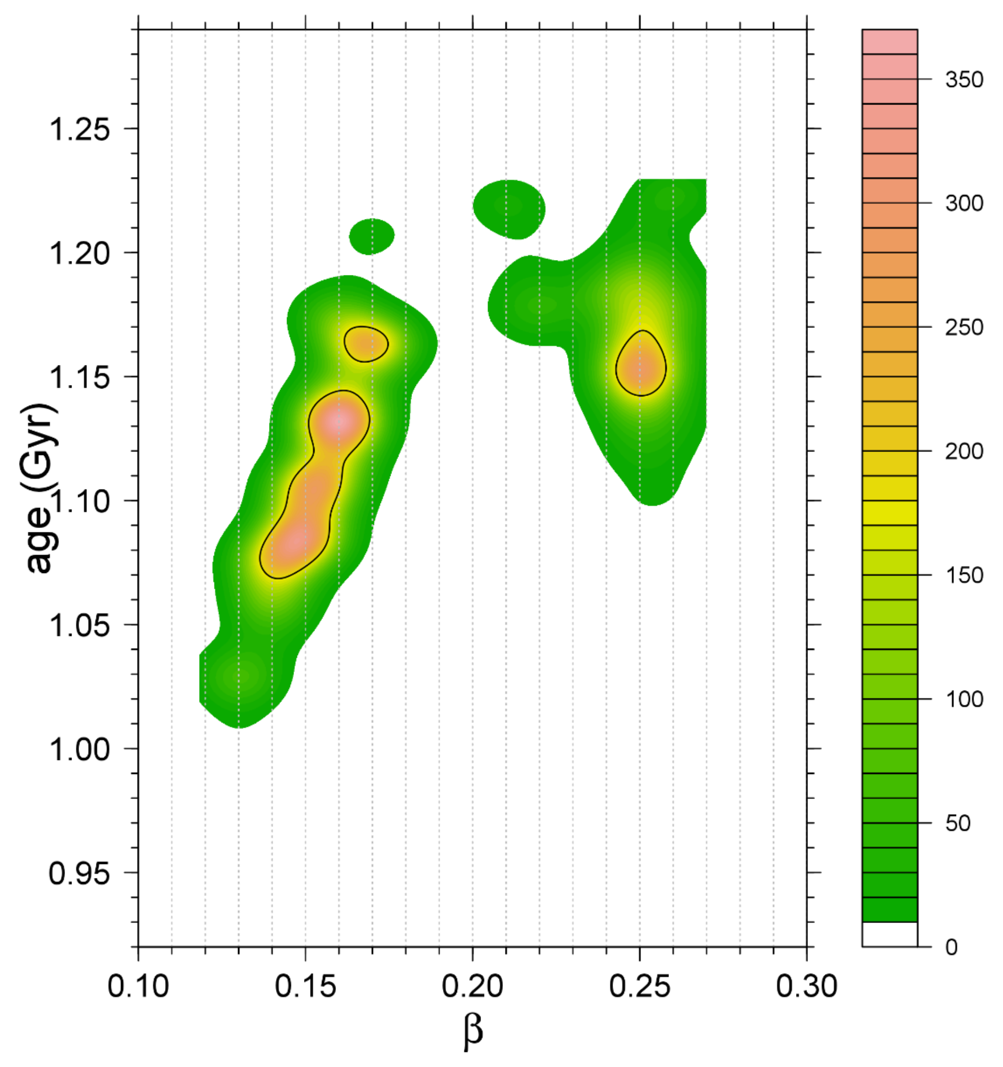


Table 1. Observational constraints for the TZ Fornacis binary system from Gallenne et al. (2016), but with stellar radii from Andersen (1991).

	primary	secondary
M ($M_\odot$)	2.057 ± 0.001	1.958 ± 0.001
R ($R_\odot$)	8.32 ± 0.12	3.96 ± 0.09
$T_{\rm eff}$ (K)	4930 ± 30	6650 ± 200
[Fe/H]	0.02 ± 0.05	-0.05 ± 0.1

Primary star in the central He-burning phase, secondary in the sub-giant branch or earlier

EB TZ Fornacis



FRANEC

Valle et al. 2017

EB TZ Fornacis

Valle et al. 2017 :

1 class

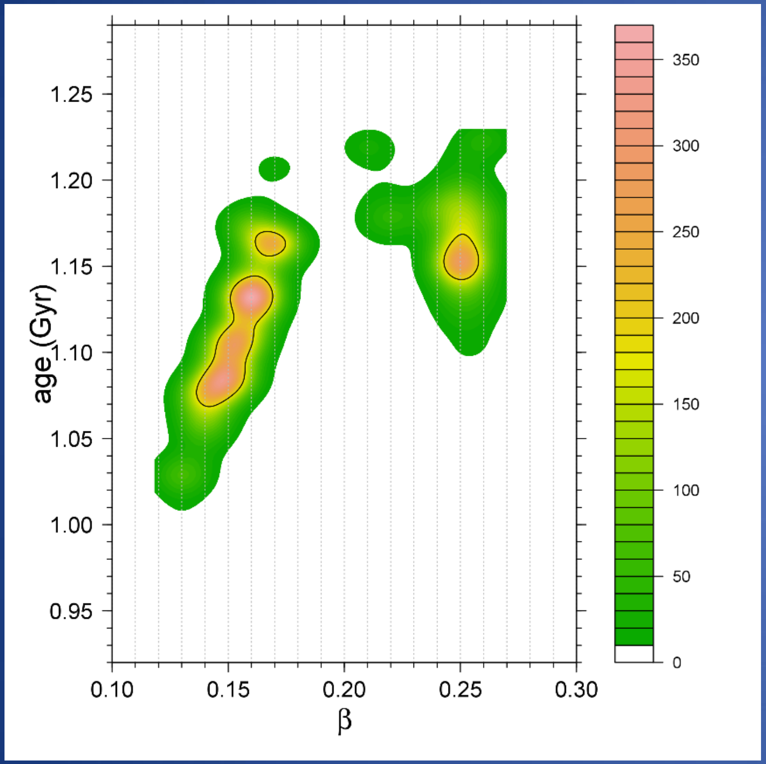
- Age: $1.11^{+0.05}_{-0.03}$Gyr
- Y=0.262 ± 0.01
- β= 0.15 ± 0.01

2 class

- Age: $1.16^{+0.03}_{-0.02}$Gyr
- Y=0.263 ± 0.001
- β= $0.25^{+0.005}_{-0.01}$

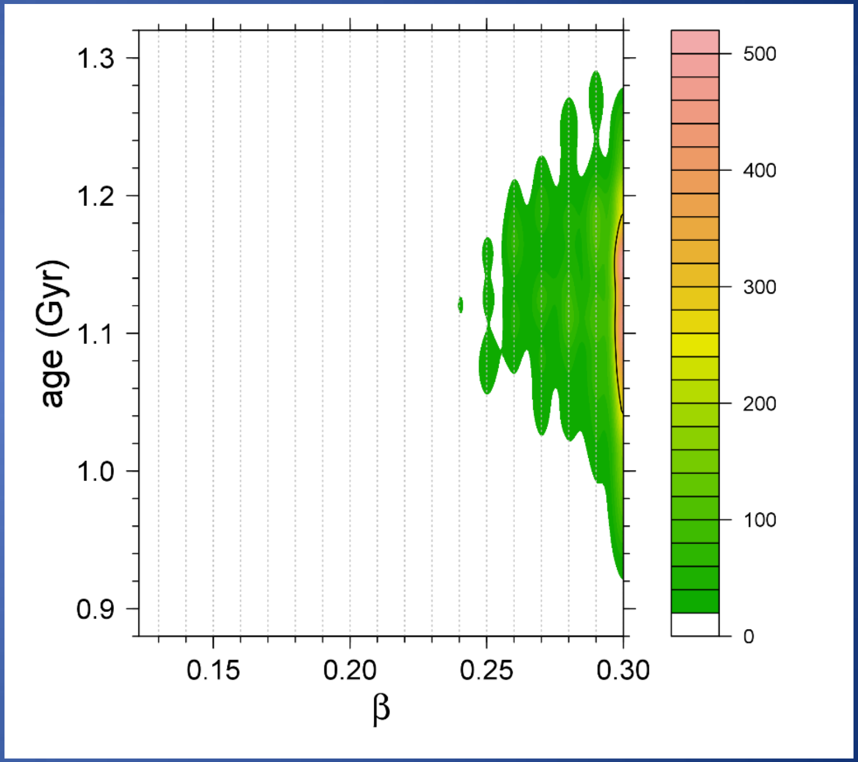
(see also Andersen et al. 1991; Pols et al. 1997; Stancliffe et al. 2015; Higl & Weiss 2017; Claret & Torres 2018)

EB TZ Fornacis: effect of mass uncertainty



$\sigma(M_1) = \sigma(M_2) = 0.05\%$

(Galenne et al 2016)

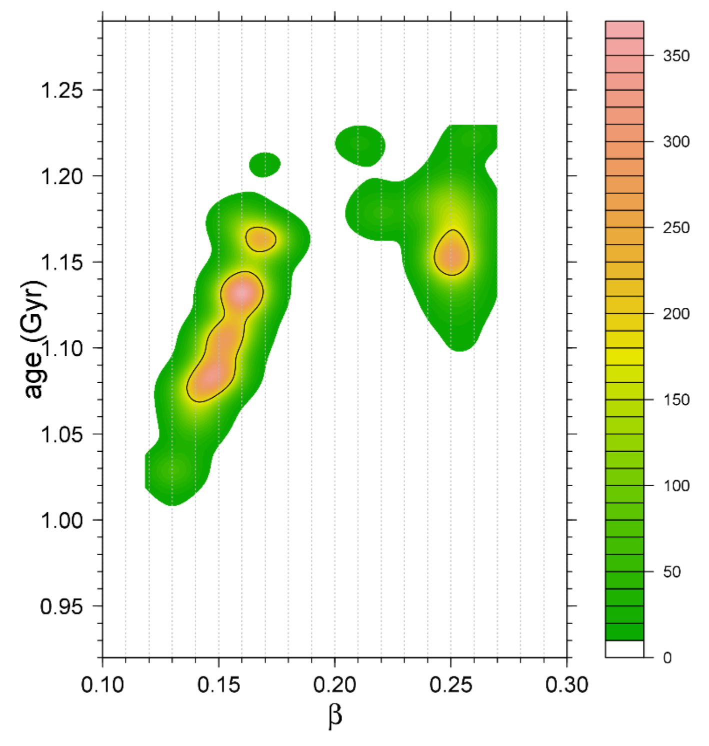


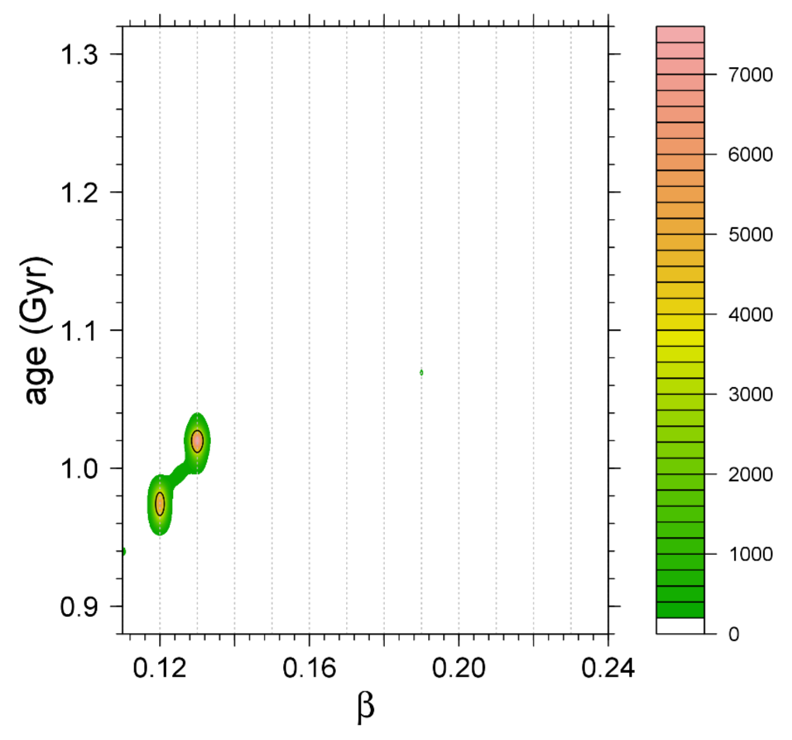
$\sigma(M_1) = 3\%$, $\sigma(M_2) = 1.5\%$

(Andersen 1991)

Valle et al. 2017

EB TZ Fornacis:
effect of helium uncertainty ΔY/ΔZ





ΔY/ΔZ variable in the range 1-3

ΔY/ΔZ = 2 fixed

Valle et al. 2017

- The calibration of stellar parameters from binary stars is affected by the priors adopted in the fitting procedure

The overshooting parameter calibrated with observations depends on:

- The overshooting scheme adopted in the code

- The input physics/parameters adopted in the code

Convection

One of the major and long-standing weaknesses in stellar models

Stellar models are not yet able to accurately predict:

- the extension of convective regions
- the temperature gradient

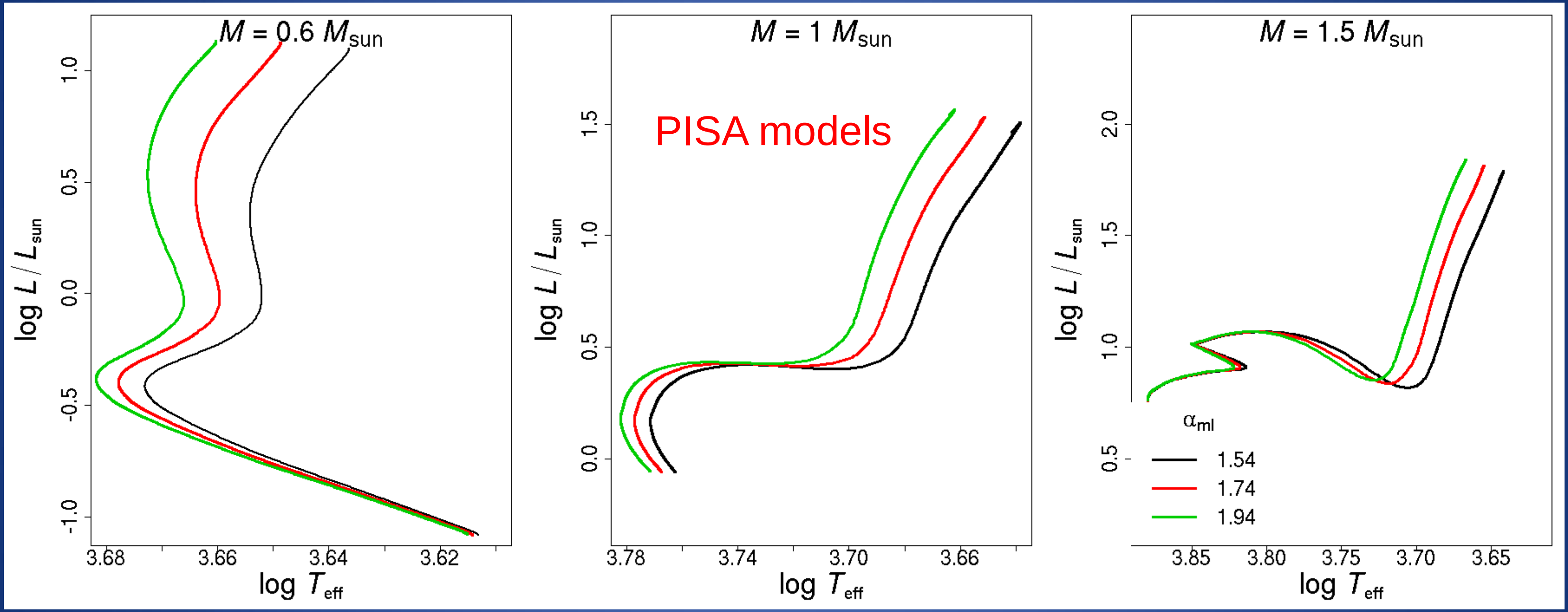
Superadiabatic convection

- In stellar codes is usually adopted the mixing-length theory *(Bierman 1932; Böhm-Vitense 1958)* to compute the temperature gradient in the outer convective regions

- $L_{ml} = \alpha H_p$, where α is a free parameter

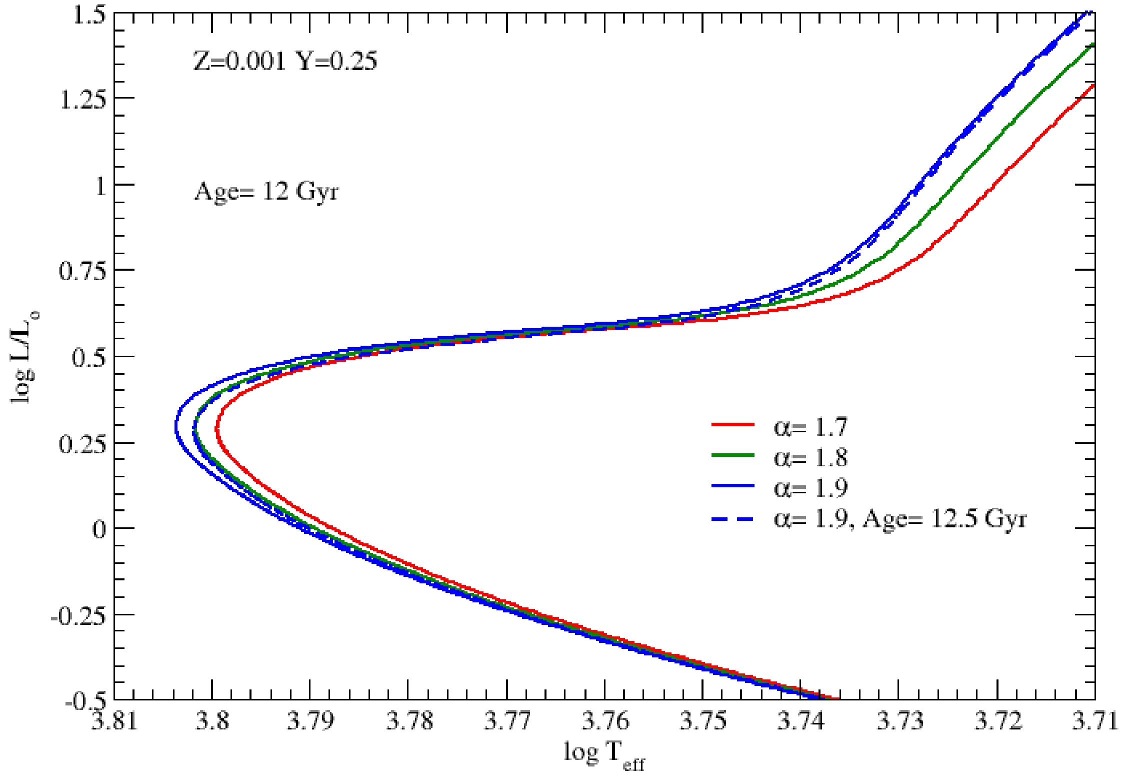
- T_{eff} of stars with a convective envelope can not be firmly predicted, because it depends on the free parameter α

Superadiabatic convection



$0.7\ M_o < M < 1.4\ M_o$: maximum impact of mixing-length in MS

Superadiabatic convection



$\Delta\alpha = 0.1$

↓

$\Delta T_{eff} \approx 30$ K

↓

Δage=0.5 Gyr

Pisa models

See also discussion in Chaboyer et al. 1998; Castellani et al. 1999; Lebreton et al. 2014

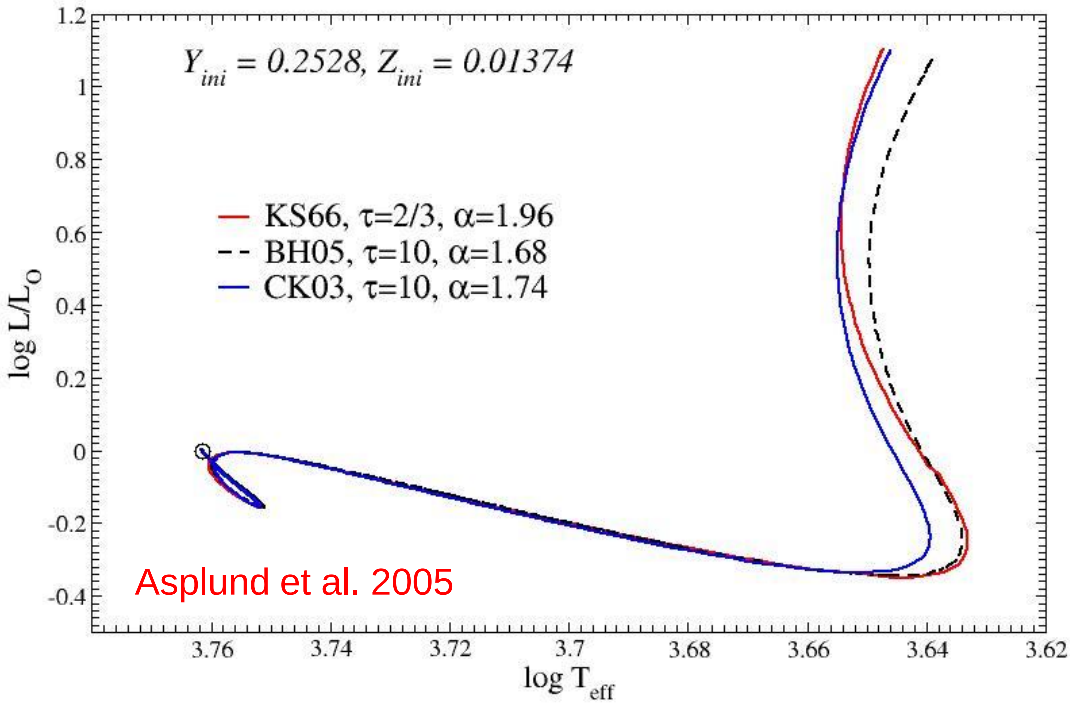
Solar calibration of α

- 1 M_o that at the age of the Sun has L_o, R_o and $(Z/X)_o$

One should remember that the solar calibrated α:

- is not necessarily suitable for stars of different masses and/or in different evolutionary phases *(Ludwig et al. 1999; Freytag et al. 1999; Trampedach et al. 2014; Magic et al. 2015; Salaris & Cassisi 2015)*
- depends on the input physics and parameters adopted in the stellar code

Solar calibration of α: different outer boundary conditions



Asplund et al. 2005

Tognelli, Prada Moroni, Degl'Innocenti 2011

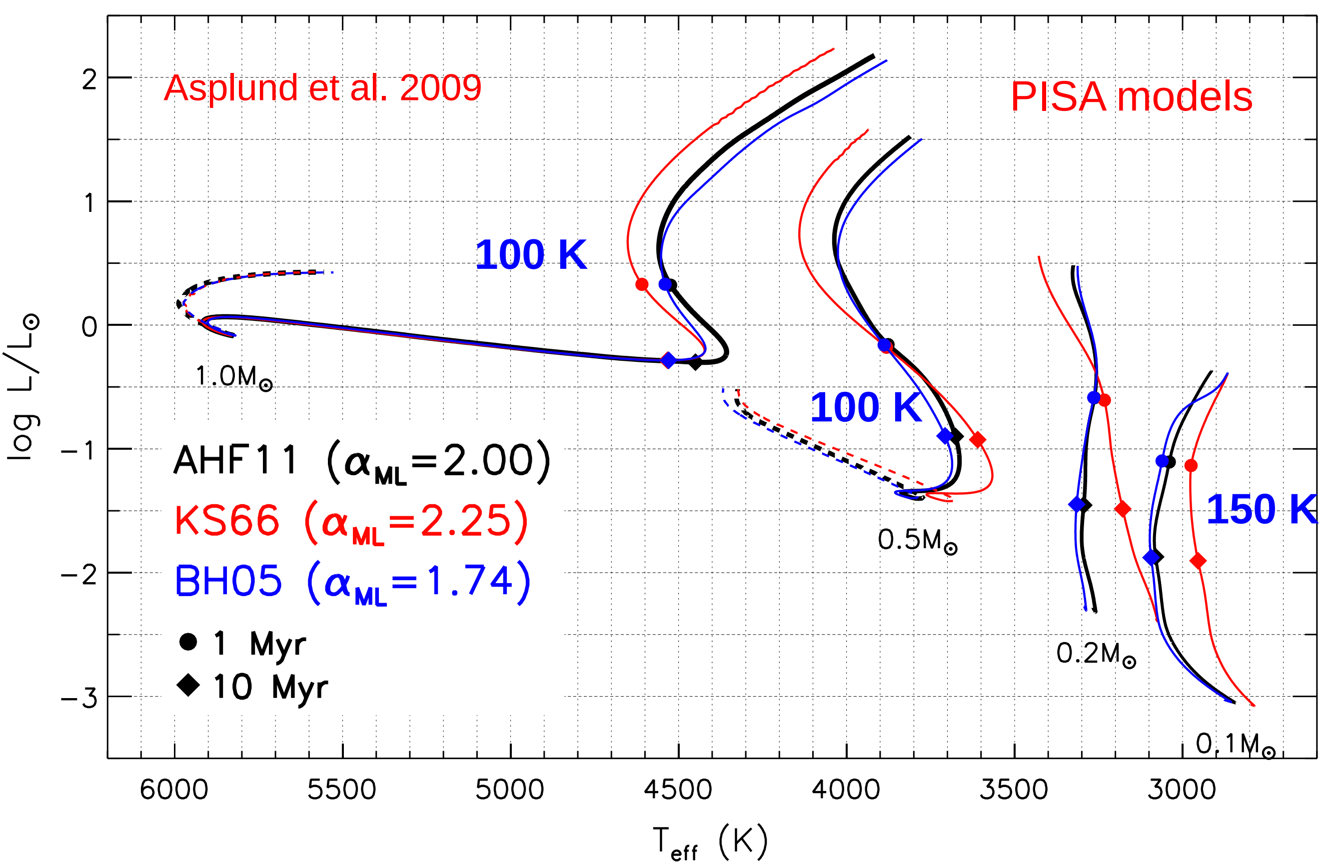
BH05: Brott & Hauschildt 2005

K66: Krishna Swamy 1966

CK03: Castelli & Kurucz 2003

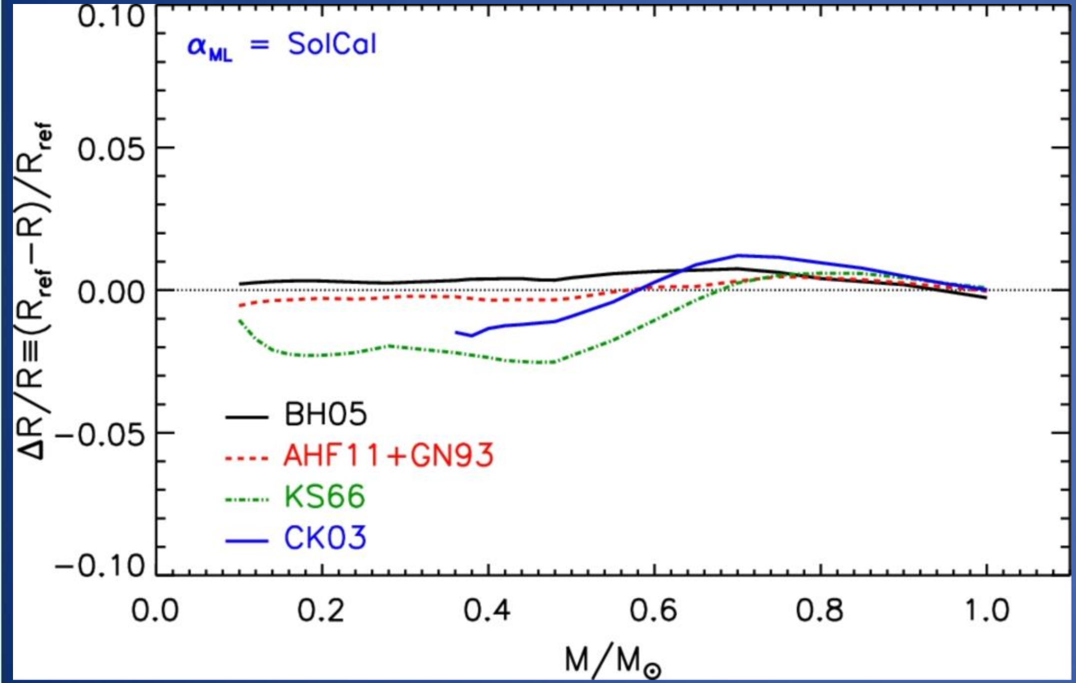
See e.g. Montalban et al. 2001, 2004; Salaris et al. 2002; Tognelli et al. 2011; Tanner et al. 2014; Salaris & Cassisi 2015

Solar calibration of α: different outer boundary conditions



Solar calibrated models that adopt different input physics and/or boundary conditions provide different T_{eff} for masses and / or evolutionary phases different from the Sun (*Salaris et al. 2002*)

Solar calibration of α: different outer boundary conditions



Tognelli, Prada Moroni, Degl'Innocenti 2018

BH05: Brott & Hauschildt 2005

AHF11: Allard et al. 2011

K66: Krishna Swamy 1966

CK03: Castelli & Kurucz 2003

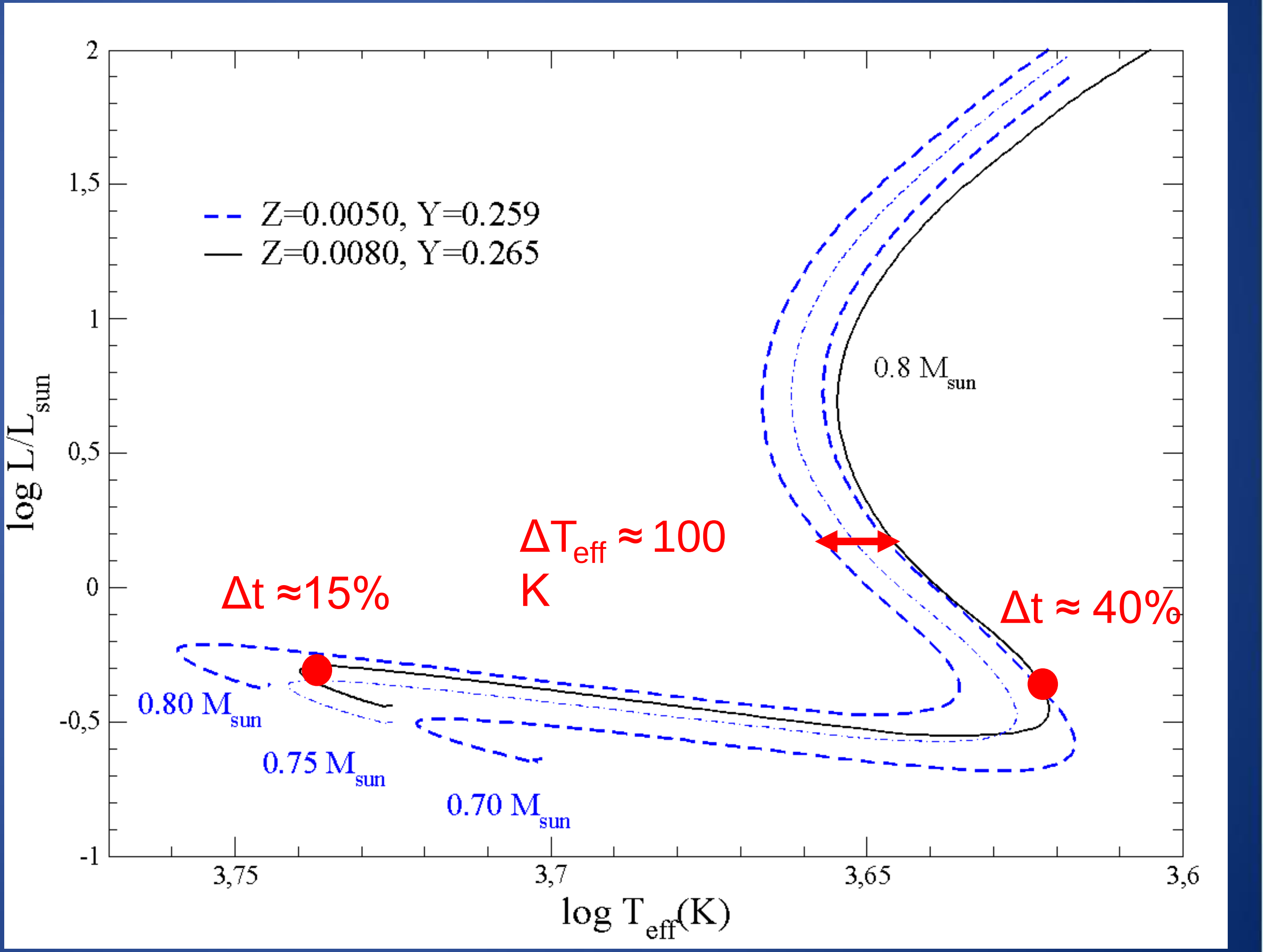
Thanks

Pre-MS tracks

Δ[Fe/H]=0.2 dex leads to a shift in T_{eff} of ≈100K

ΔM=0.1 M_o

Δt≈40 %



Initial chemical abundance

- The initial Y, Z, and element mixture required in stellar model computations rely on some assumption

$$Z = \frac{(1 - Y_P)(Z/X)_\odot}{10^{-[Fe/H]} + (1 + \Delta Y/\Delta Z)(Z/X)_\odot}$$

$$Y = Z\frac{\Delta Y}{\Delta Z} + Y_P$$

- Not negligible uncertainty

- How do these uncertainties propagate into model predictions?

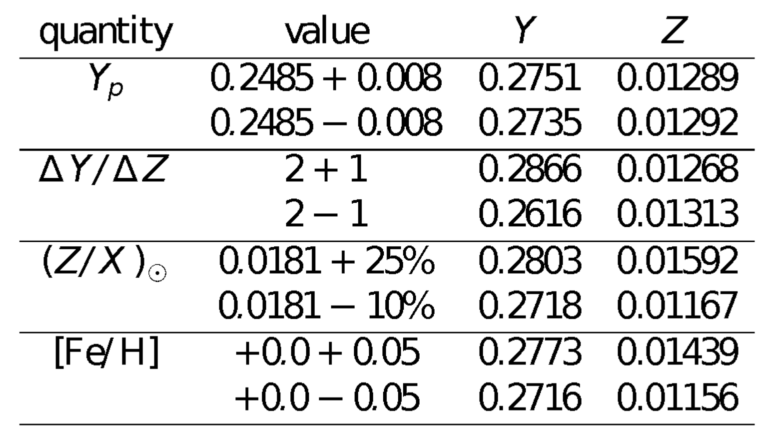
Uncertainty in initial chemical composition

Reference value:

Y=0.274, Z=0.01291, [Fe/H]=0

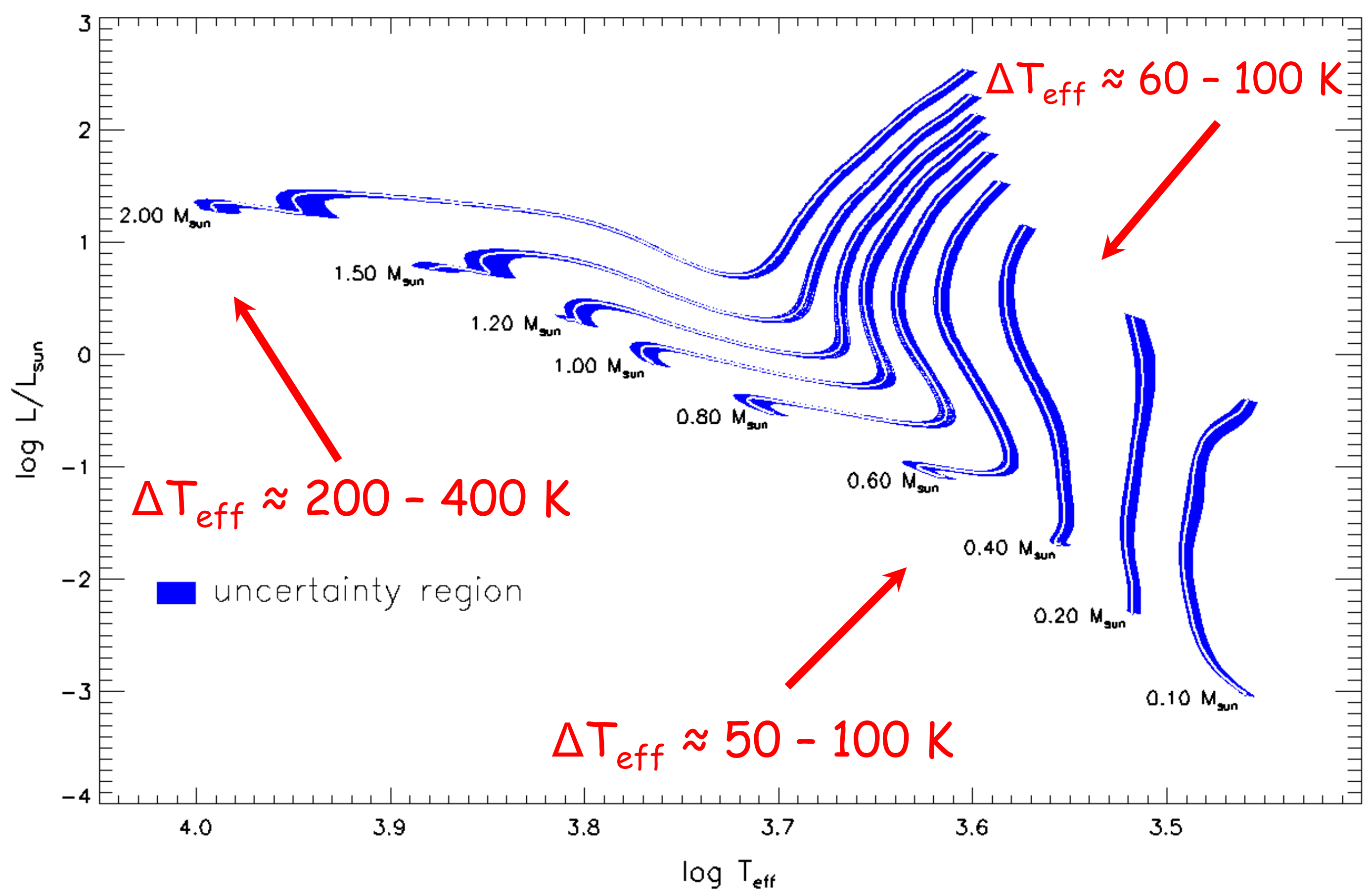
Variation range:

- Δ[Fe/H]= ±0.05
- ΔY_P = ±0.008 (Cyburt 2004)
- $\Delta Y/\Delta Z$ = 2 ± 1 (Casagrande 2007, Gennaro et al. 2010)
- $\Delta(Z/X)_{sun}$ ≈ +25/-10 % (Tognelli et al. 2012)



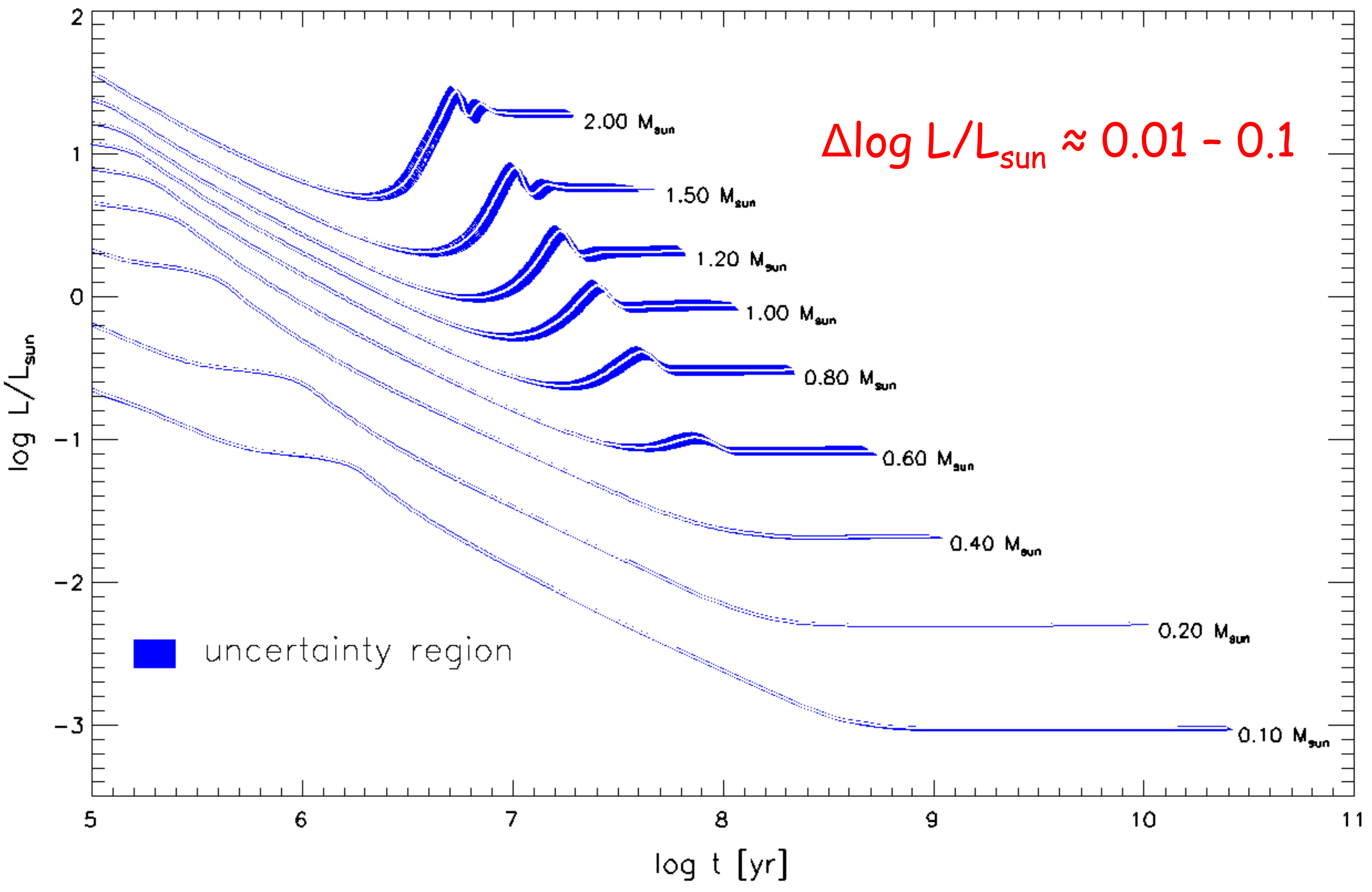
quantity	value	Y	Z
Y_p	$0.2485 + 0.008$	0.2751	0.01289
	$0.2485 - 0.008$	0.2735	0.01292
$\Delta Y/\Delta Z$	$2 + 1$	0.2866	0.01268
	$2 - 1$	0.2616	0.01313
$(Z/X)_\odot$	$0.0181 + 25\%$	0.2803	0.01592
	$0.0181 - 10\%$	0.2718	0.01167
[Fe/H]	$+0.0 + 0.05$	0.2773	0.01439
	$+0.0 - 0.05$	0.2716	0.01156

Cumulative error due to the uncertainty in initial chemical composition



Tognelli (2013)

Cumulative error due to the uncertainty in initial chemical composition



$\Delta \log L/L_{sun} \approx 0.01 - 0.1$

Tognelli (2013)

Helium abundance

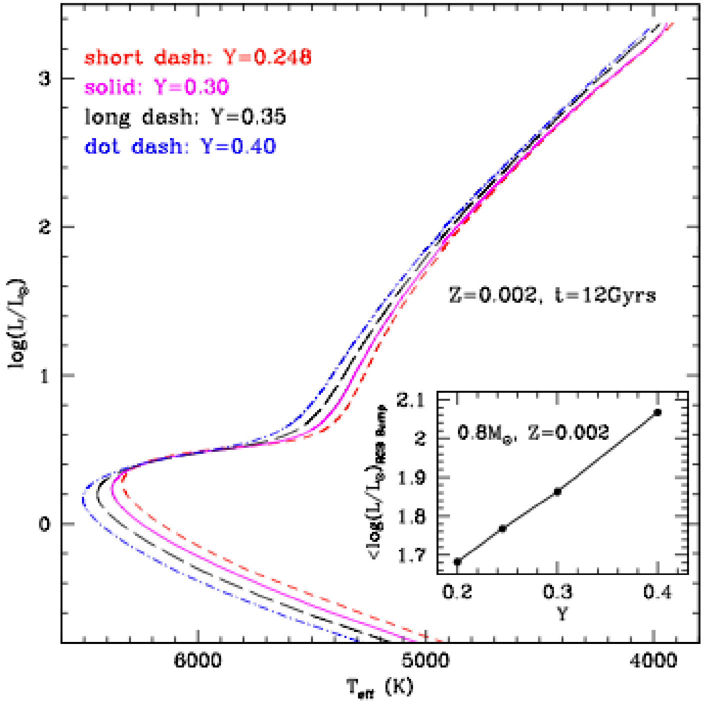
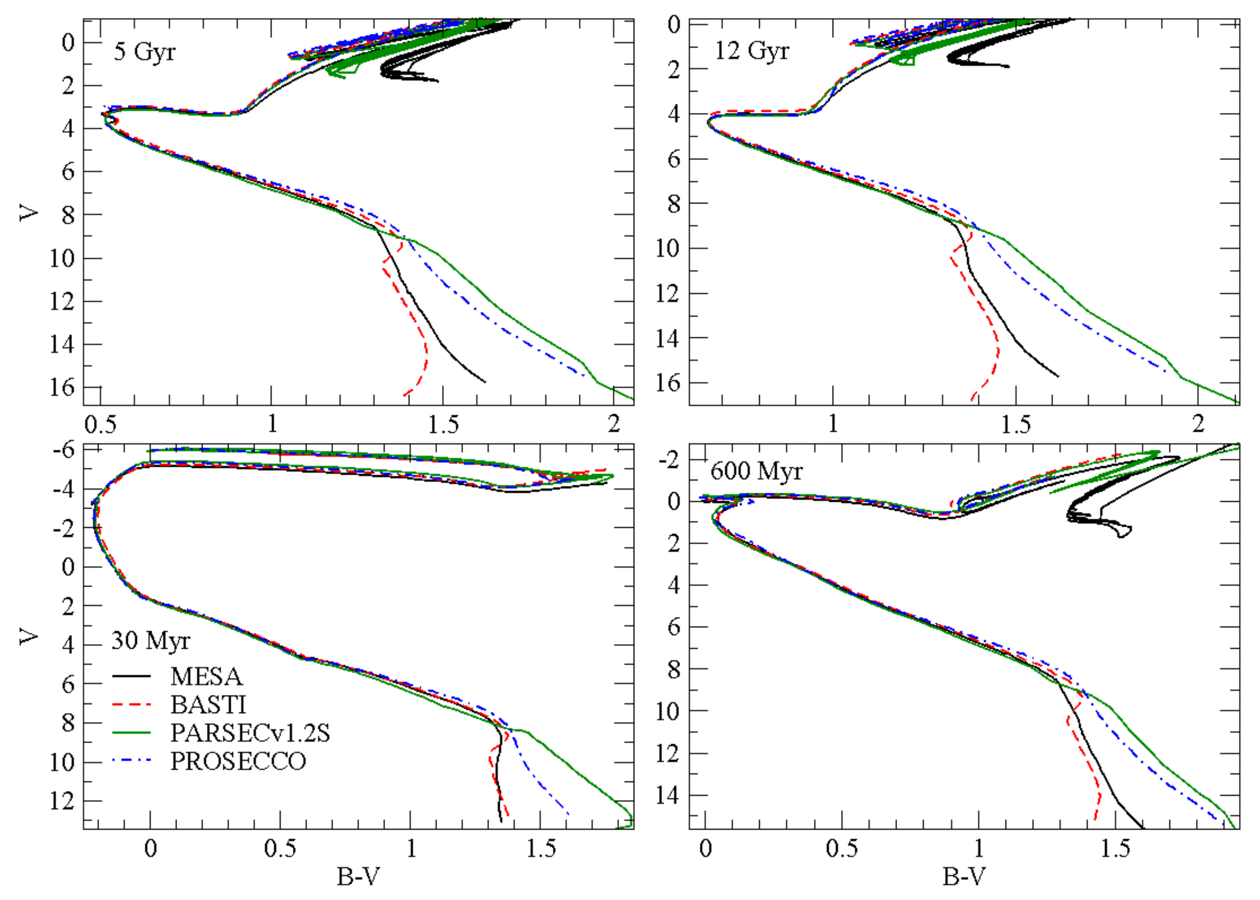


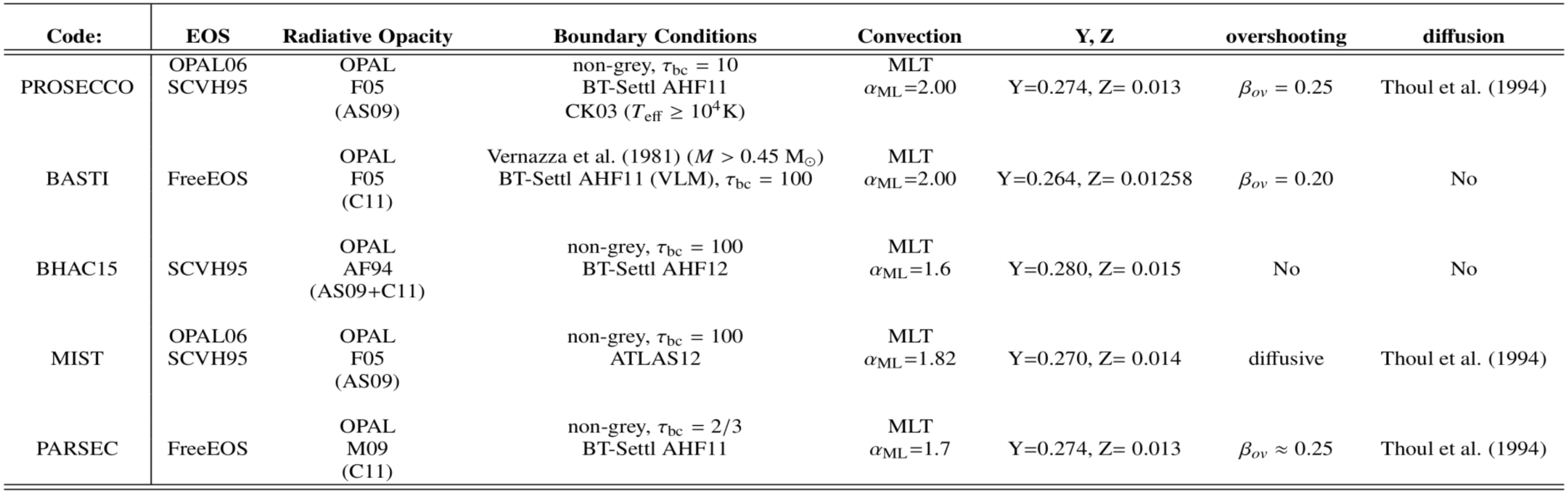
Fig. 13. Comparison between 12 Gyr old, Z=0.002, isochrones computed for various assumptions about the initial He abundance. The inset shows the trend of the average RGB bump brightness as a function of the initial He abundance for a 0.8$M_\odot$ model.

Cassisi (2013)

Comparing different models



Comparing different models

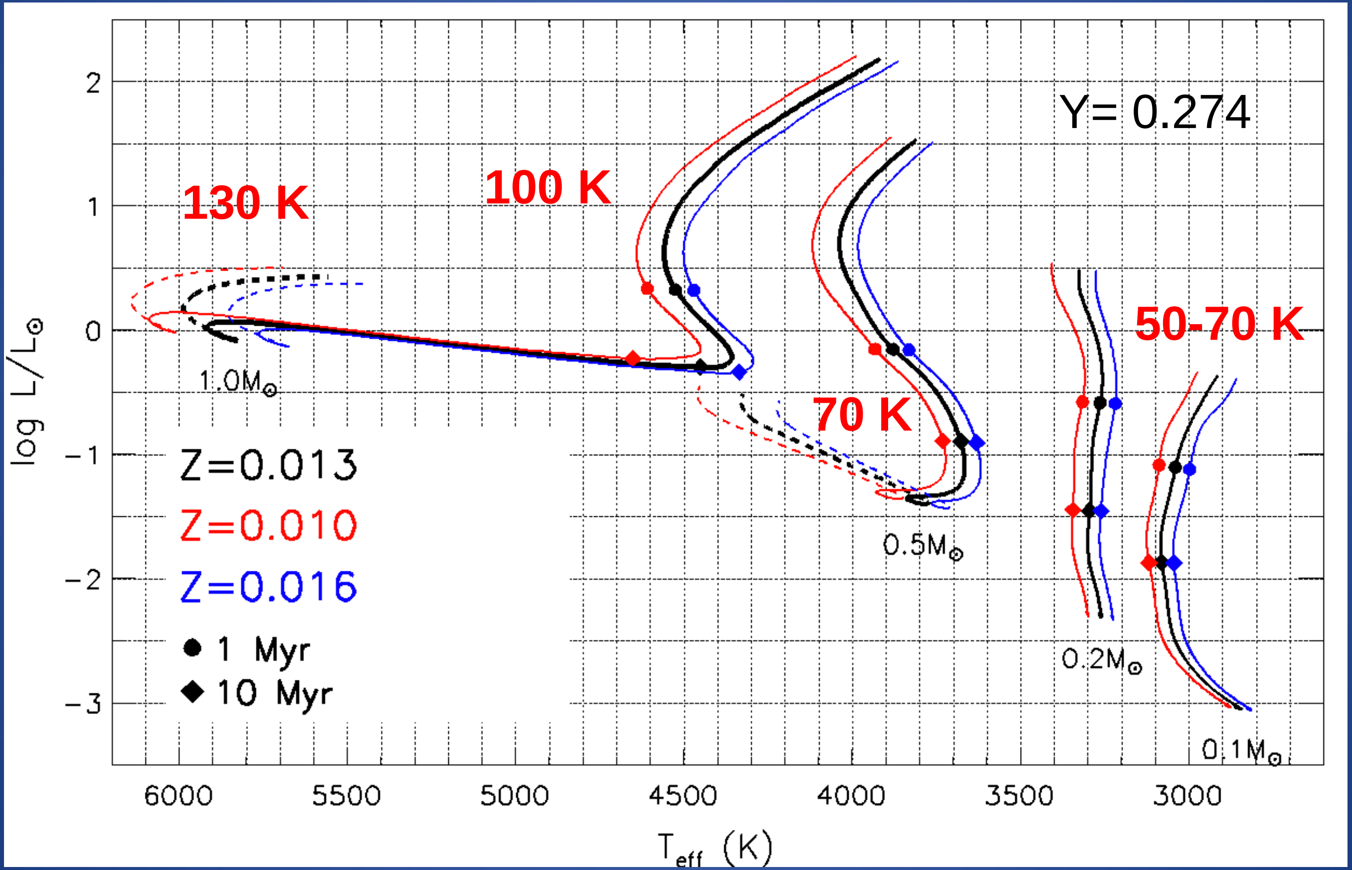


Code:	EOS	Radiative Opacity	Boundary Conditions	Convection	Y, Z	overshooting	diffusion
PROSECCO	OPAL06 SCVH95	OPAL F05 (AS09)	non-grey, $\tau_{\rm bc} = 10$ BT-Settl AHF11 CK03 ($T_{\rm eff} \geq 10^4$K)	MLT $\alpha_{\rm ML}$=2.00	Y=0.274, Z= 0.013	$\beta_{ov} = 0.25$	Thoul et al. (1994)
BASTI	FreeEOS	OPAL F05 (C11)	Vernazza et al. (1981) ($M > 0.45$ M$_\odot$) BT-Settl AHF11 (VLM), $\tau_{\rm bc} = 100$	MLT $\alpha_{\rm ML}$=2.00	Y=0.264, Z= 0.01258	$\beta_{ov} = 0.20$	No
BHAC15	SCVH95	OPAL AF94 (AS09+C11)	non-grey, $\tau_{\rm bc} = 100$ BT-Settl AHF12	MLT $\alpha_{\rm ML}$=1.6	Y=0.280, Z= 0.015	No	No
MIST	OPAL06 SCVH95	OPAL F05 (AS09)	non-grey, $\tau_{\rm bc} = 100$ ATLAS12	MLT $\alpha_{\rm ML}$=1.82	Y=0.270, Z= 0.014	diffusive	Thoul et al. (1994)
PARSEC	FreeEOS	OPAL M09 (C11)	non-grey, $\tau_{\rm bc} = 2/3$ BT-Settl AHF11	MLT $\alpha_{\rm ML}$=1.7	Y=0.274, Z= 0.013	$\beta_{ov} \approx 0.25$	Thoul et al. (1994)

EB TZ Fornacis: effect of mass uncertainty

- Gallenne et al. 2016: $\sigma(M_1)$= $\sigma(M_1)$= 0.001 M_o (0.05%)
- Andersen 1991: $\sigma(M_1)$= 0.06 M_o (3%), $\sigma(M_2)$=0.03 M_o (1.5%)
- $\sigma(M) \approx 1$ % are common
- What’s the effect on overshooting calibration of increasing the mass uncertainty?

Varying Z (Δ[Fe/H]= ± 0.1), keeping fixed Y

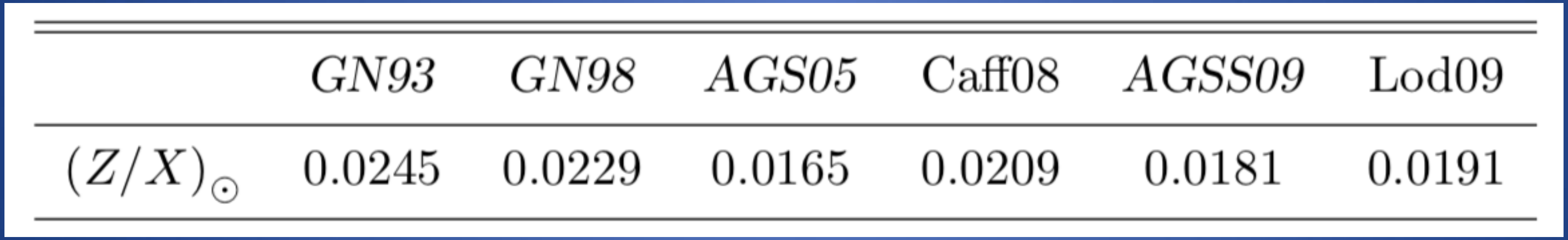


PISA models

Solar element mixture

- In the last 25 years it has been revised several times

- GN93, Grevesse & Noels 1993; GS98, Grevesse & Sauval 1998; AGS05, Asplund et al. 2005; Caff08, Caffau et al. 2008; AGSS09, Asplund et al. 2009; Lod09, Lodders et al. 2009



	GN93	GN98	AGS05	Caff08	AGSS09	Lod09
$(Z/X)_\odot$	0.0245	0.0229	0.0165	0.0209	0.0181	0.0191

Lebreton et al. 2009

From GN93 to AGSS09 a decrease of:

- 34% of ^{16}O abundance
- 25% of $(Z/X)_o$

Varying the element mixture

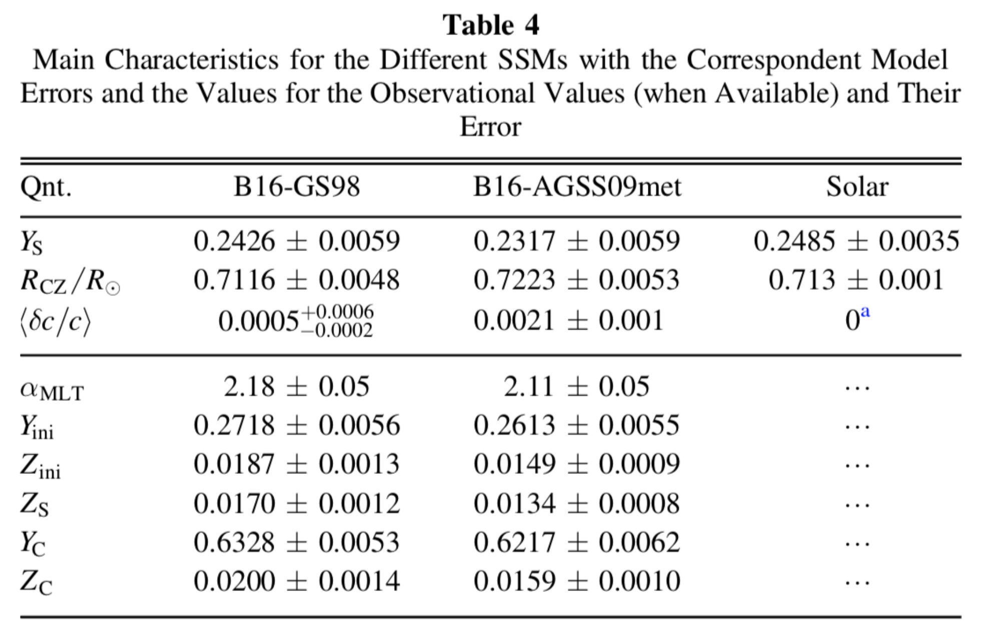


Table 4

Main Characteristics for the Different SSMs with the Correspondent Model Errors and the Values for the Observational Values (when Available) and Their Error

Qnt.	B16-GS98	B16-AGSS09met	Solar
Y_S	0.2426 ± 0.0059	0.2317 ± 0.0059	0.2485 ± 0.0035
$R_{CZ}/R_\odot$	0.7116 ± 0.0048	0.7223 ± 0.0053	0.713 ± 0.001
$\langle \delta c/c \rangle$	$0.0005^{+0.0006}_{-0.0002}$	0.0021 ± 0.001	0[a]
α_{MLT}	2.18 ± 0.05	2.11 ± 0.05	…
Y_{ini}	0.2718 ± 0.0056	0.2613 ± 0.0055	…
Z_{ini}	0.0187 ± 0.0013	0.0149 ± 0.0009	…
Z_S	0.0170 ± 0.0012	0.0134 ± 0.0008	…
Y_C	0.6328 ± 0.0053	0.6217 ± 0.0062	…
Z_C	0.0200 ± 0.0014	0.0159 ± 0.0010	…

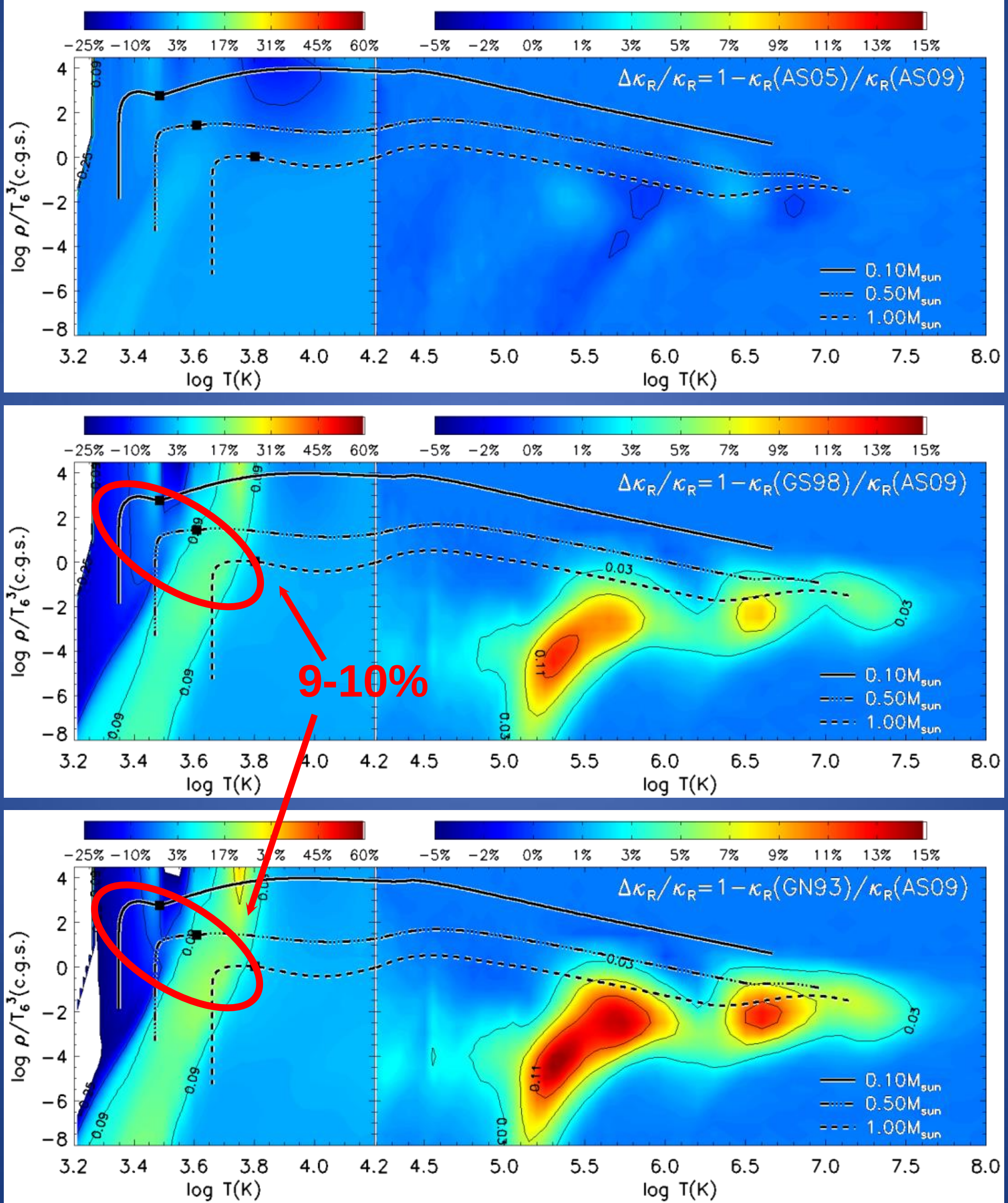
Barcelona SSM (Vinyoles et al. 2017)

- Effect on the standard solar model

From GS98 to AGSS09, a decrease in:

- Z_{ini}:20 %
- Y_{ini}: 4 %
- ΔY/ΔZ from 1.3 to 0.845

Varying the element mixture

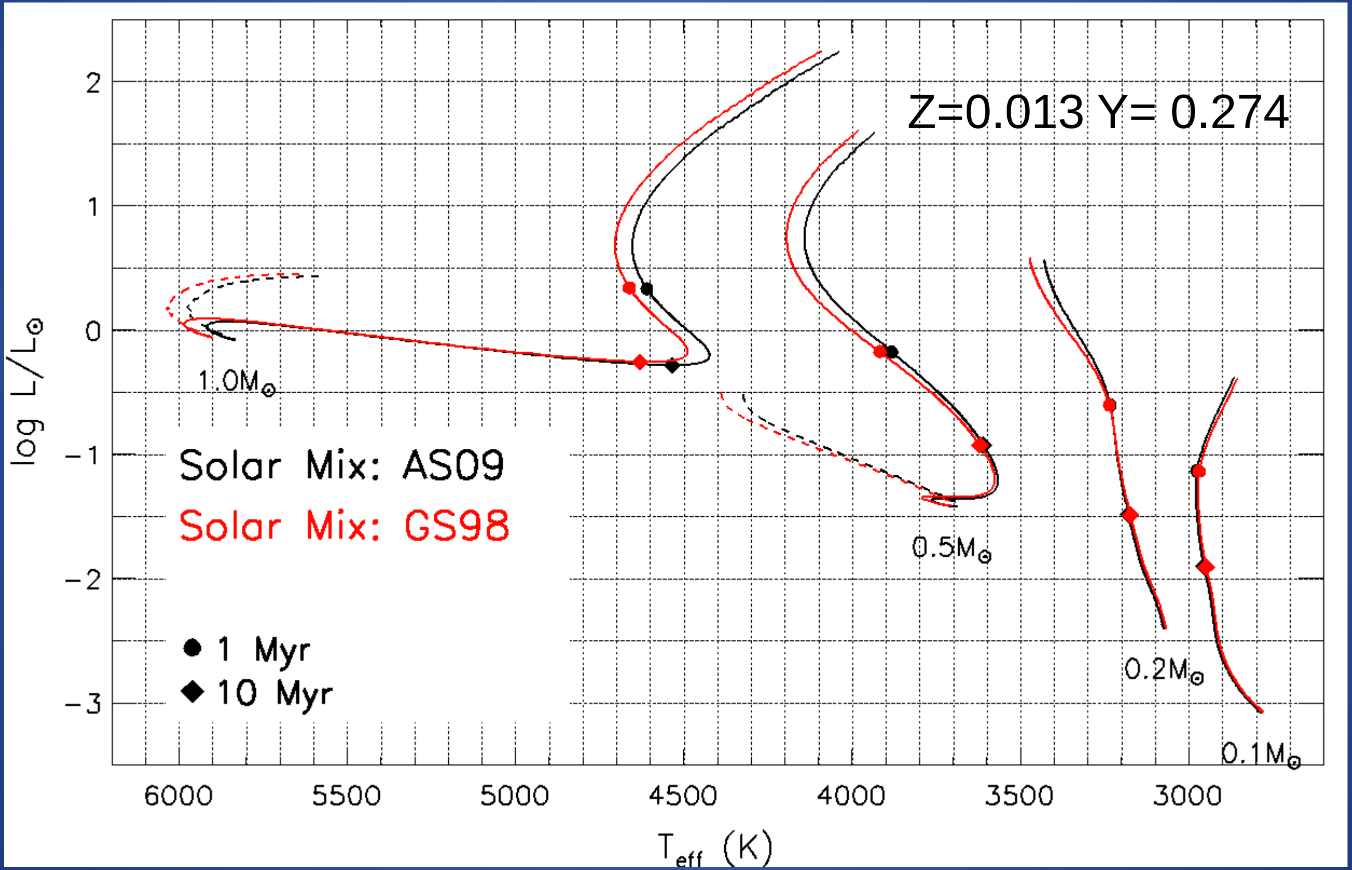


Tognelli (2013)

$1M_{sun}$, 3-8%

$1M_{sun}$, 7-10%

Varying the element mixture



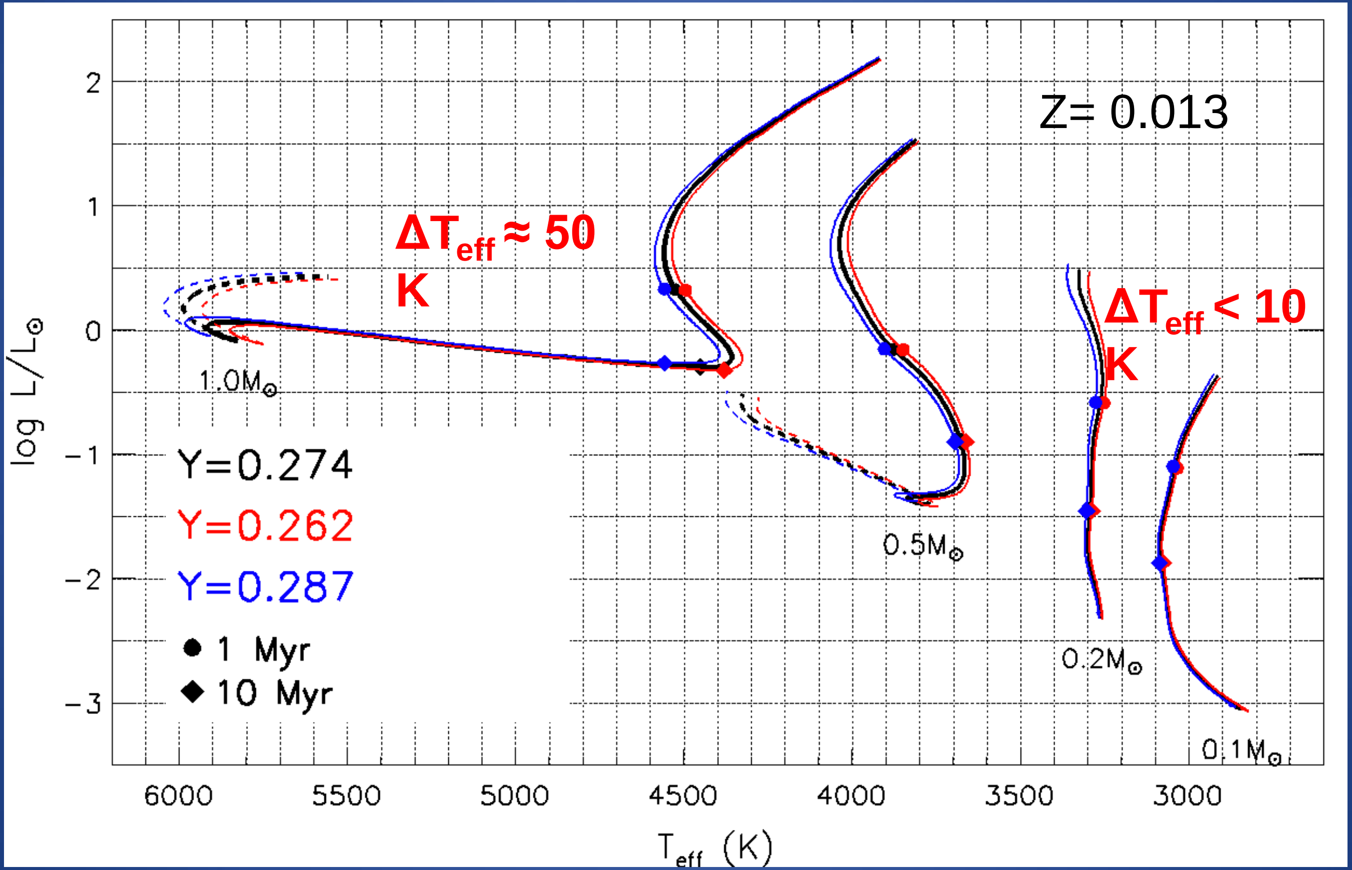
PISA models

Initial He abundance uncertainty

$$Y = Y_p + \frac{\Delta Y}{\Delta Z} Z$$

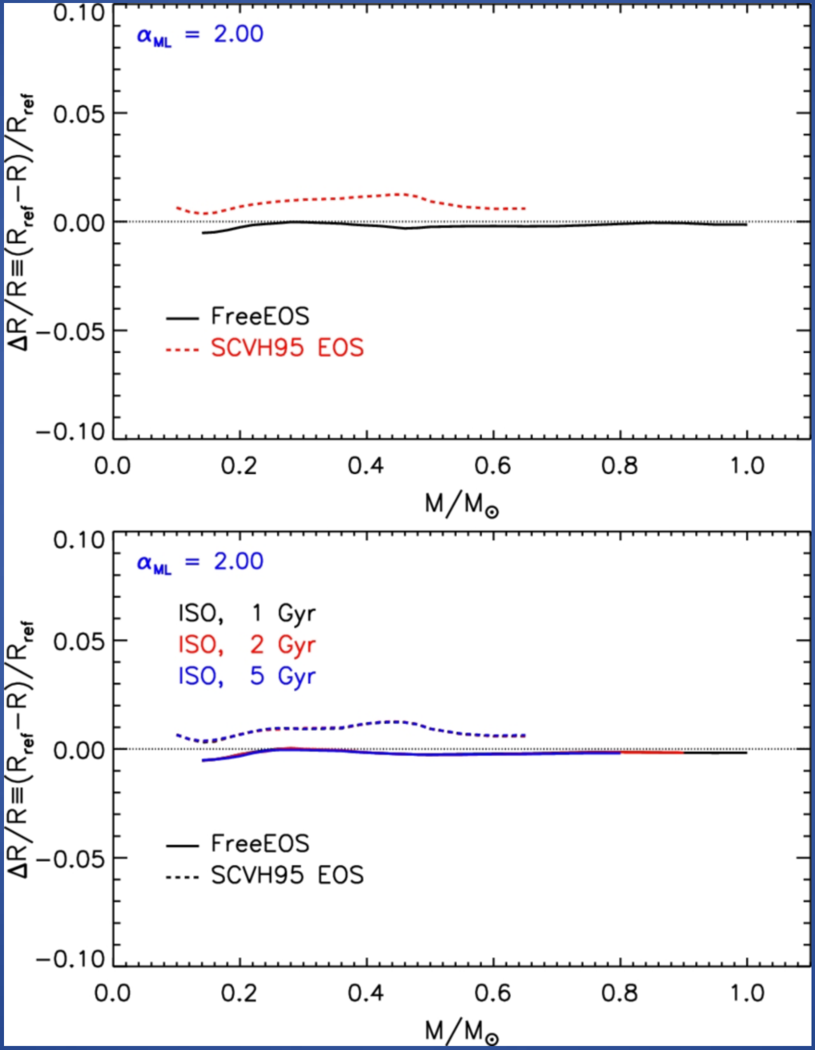
- Y_p well constrained: 0.2487 ± 0.002 Cyburt et al. 2008 (WMAP); 0.2463 ± 0.003 Coc et al. 2013 (Planck)
- $\Delta Y/\Delta Z$ largely uncertain: 2 ± 1 (Pagel & Portinari 1998, Casagrande et al. 2007; Gennaro et al. 2010)

Varying Y (ΔY/ΔZ ± 1), keeping fixed Z



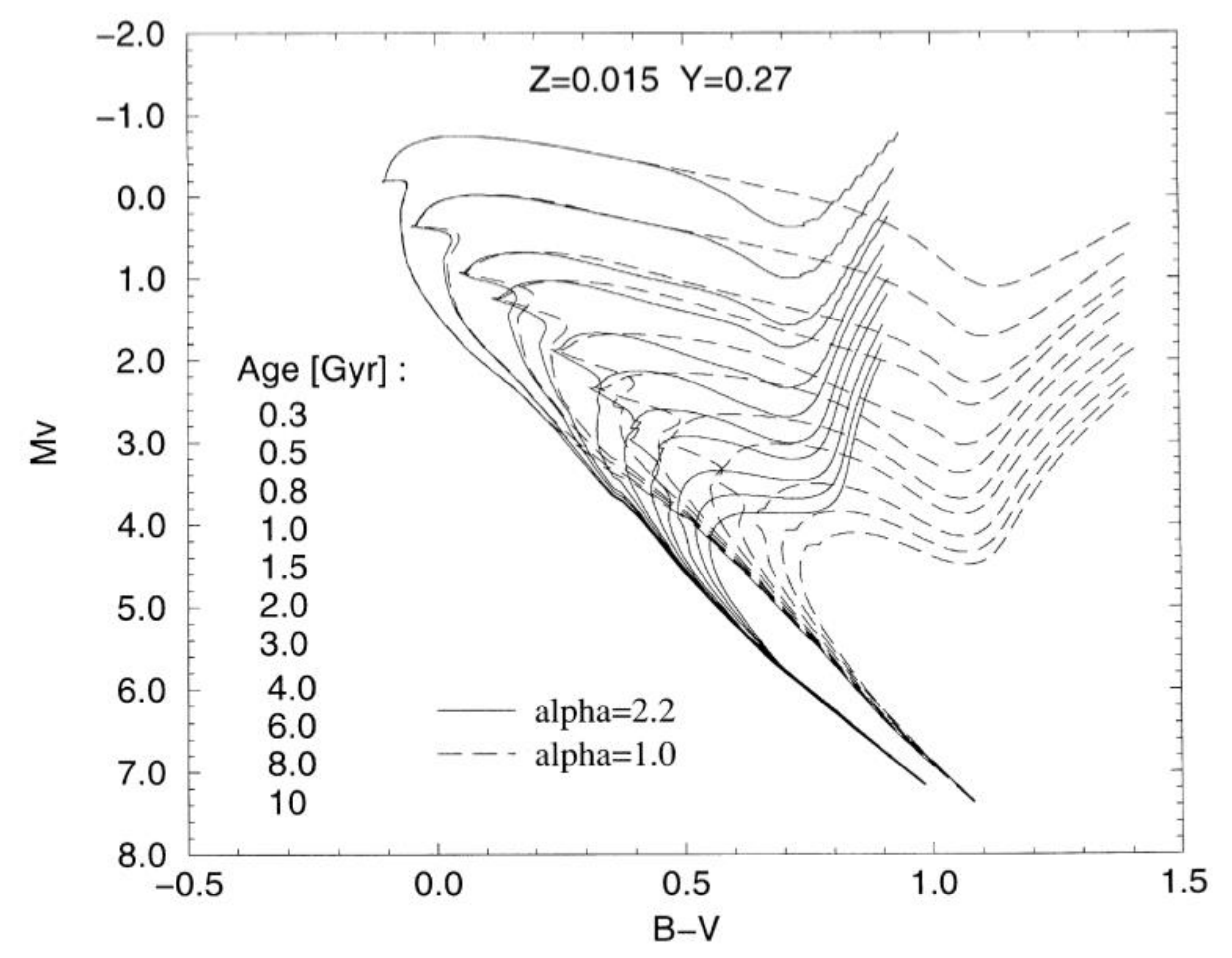
PISA models

Different EOSs



Tognelli, Prada Moroni, Degl’Innocenti 2018

Superadiabatic convection



Castellani et al. 1999