Challenges for fitting observations of PLATO stars on the main sequence

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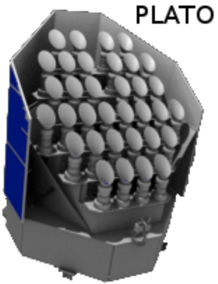
Introduction

Observational context

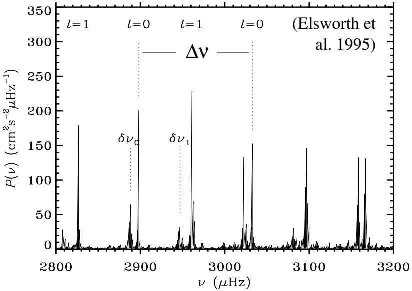
- a large number of main-sequence pulsators to be observed by PLATO 2.0 ($\sim$ 20000)
  - two orders of magnitude more than Kepler
  - prime targets for the PLATO mission (in particular the exoplanet host stars)
- need for efficient and accurate methods for characterising these stars using asteroseismology



Solar-like pulsators

Asteroseismology

- modes are easy to identify
  - easily recognisable frequency patterns
  - regular mode amplitudes
- hence, asteroseismology in these stars is relatively straightforward



Introduction

Asteroseismic inferences

- three different methods (e.g. Gough 1985):
  - forward analysis
  - analytical methods (asymptotic approach, glitch fitting)
  - formal inversion techniques

Introduction

"Fast. Cheap. Correct. Which two would you like?"

Introduction

"Fast. Cheap. Correct. Which two would you like?"

"Fast. Precise. Accurate. Which two would you like?"

Global vs. local methods

Global methods

- genetic algorithms (AMP; Brassard, Charpinet et al.)
- MCMC (AIMS; Bazot et al. 2012)
- various forms of scanning a grid (BASTA)

Local methods

- Levenberg-Marquardt (OSM; Lebreton et al.)
- Amoeba simplex (MESA)

Global vs. local methods

Global methods

- maps entire parameter space
- robust to secondary solutions
- may be very time consuming
- what grid resolution?
- how do you handle models with missing pulsation frequencies?

Local methods

- searches a specific region
- vulnerable to secondary solutions
- may be more efficient

Global methods

Some global methods

Optim.	Model treatment	Code/article	Comp. time
χ^2 min.	scan grid		
Bayesian	scan grid	BASTA	1 min*
MCMC	interpolate in grid	AIMS	10 min - 1 hr*
Genetic	models on the fly	AMP	20 000 hrs
		Charpinet et al. (2005)	
MCMC	models on the fly	Bazot et al. (2012)	

* excluding the grid computation time

Combining local and global methods

- **idea**: apply grid-based global method to find local solutions as inputs for local methods

Combining local and global methods

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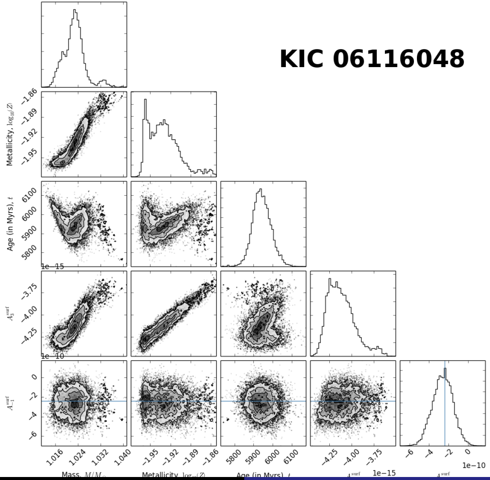
- how does one reliably identify local solutions?



Interpolation

Different options

- how to interpolate?
  - interpolate frequencies
  - interpolate model and recalculate frequencies
- how to obtain interpolation coefficients
  - Cartesian multilinear approach
  - triangulation
- age interpolation

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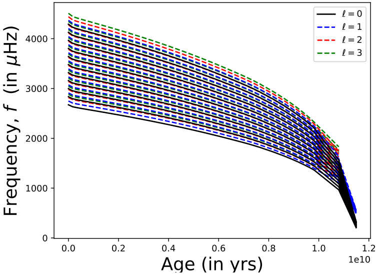
- how to keep the results self-consistent
- how to quantify interpolation errors

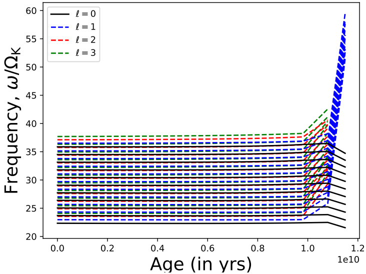
Frequency interpolation

- simplest option: interpolate frequencies directly



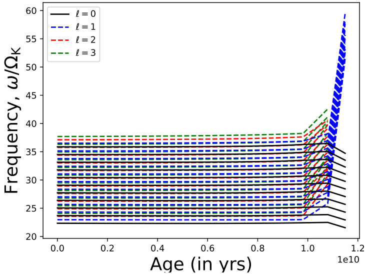
Frequency interpolation

- simplest option: interpolate frequencies directly
- better option: interpolate non-dimensional frequencies



Frequency interpolation

- simplest option: interpolate frequencies directly
- better option: interpolate non-dimensional frequencies
- this raises the question as to how to interpolate M, R



Model interpolation

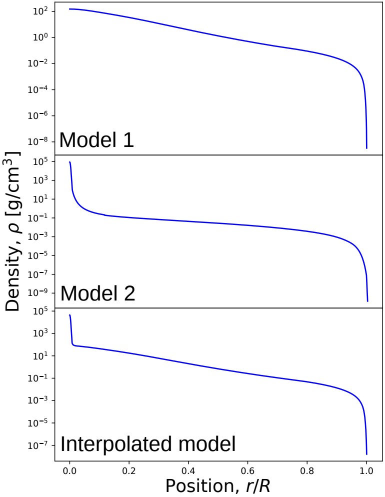
- impose the following relations

$$\begin{aligned}\rho_3(xR_3) &= a\rho_1(xR_1) + b\rho_2(xR_2)\\ \Gamma_{1,3}(xR_3) &= a\Gamma_{1,1}(xR_1) + b\Gamma_{1,2}(xR_2)\\ M_3 &= aM_1 + bM_2\end{aligned}$$

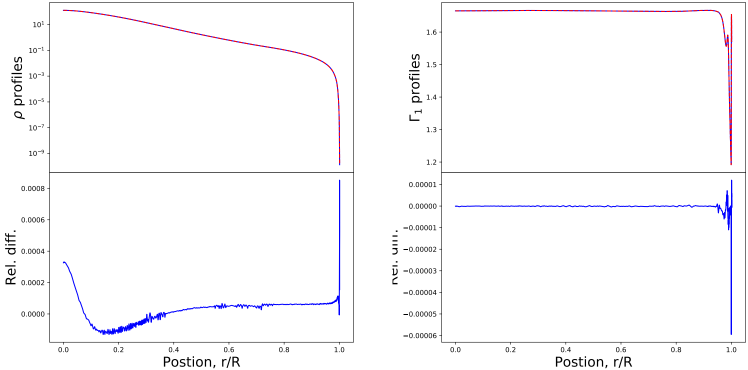
- Eqs. 1 and 3 lead to :

$$\frac{M_3}{R_3^3} = a\frac{M_1}{R_1^3} + b\frac{M_2}{R_2^3}$$

- the remaining acoustic variables (P, N^2, etc.) can be determined self-consistently

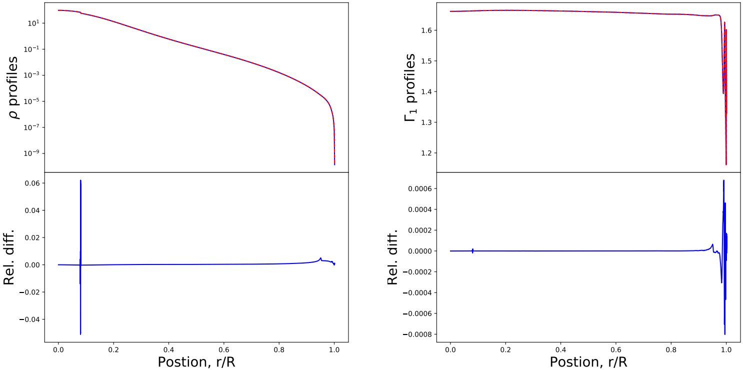


Model interpolation



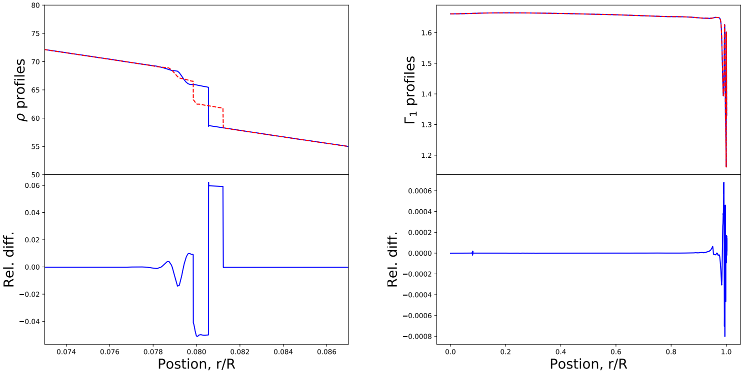
- 1 $M_\odot$

Model interpolation



- 1.5 $M_{\odot}$

Model interpolation



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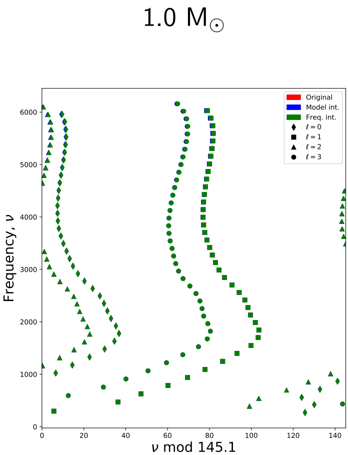
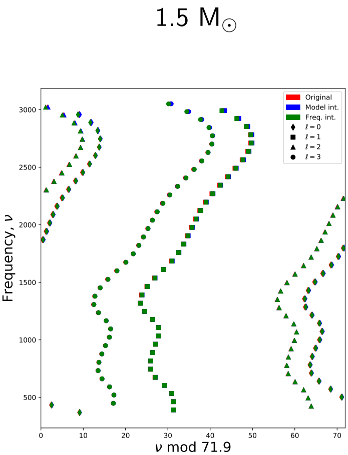
How do these different interpolations affect frequencies?





- echelle diagram of interpolated models

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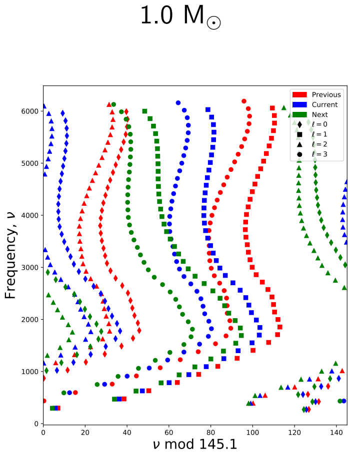

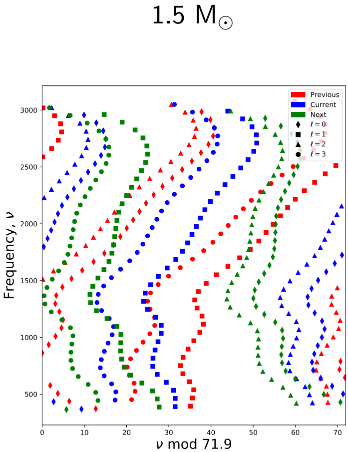

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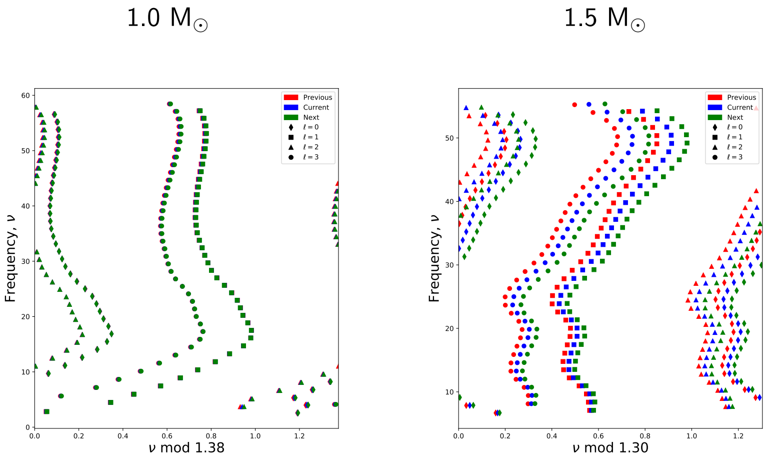

- non-dimensional echelle diagram of consecutive models

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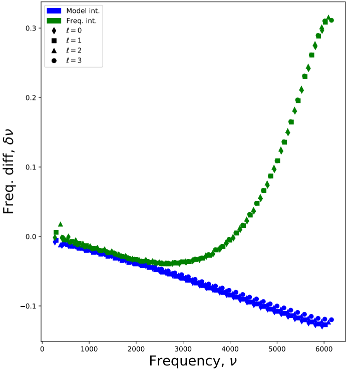

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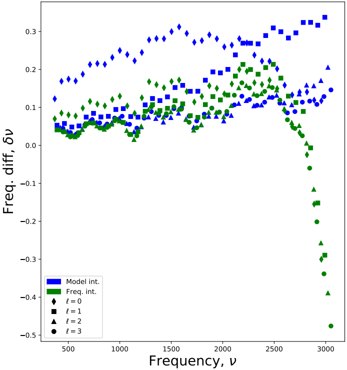
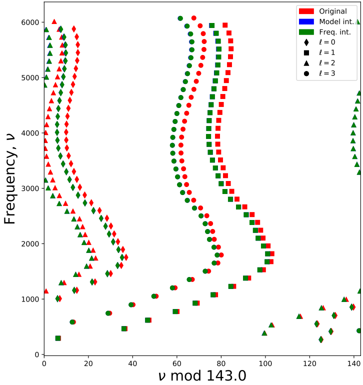


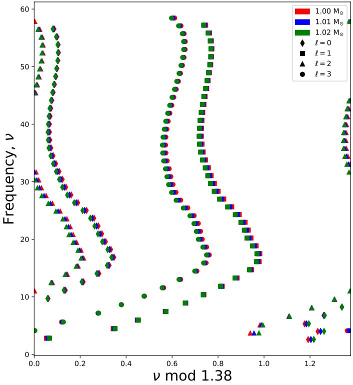
- frequency differences

Interpolation between tracks



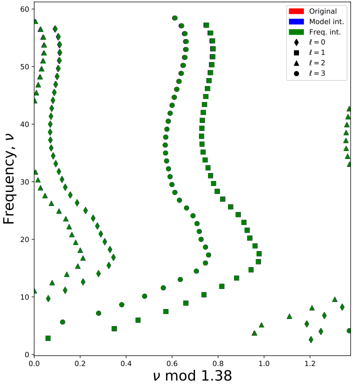
- echelle diagram of interpolated models

Interpolation between tracks



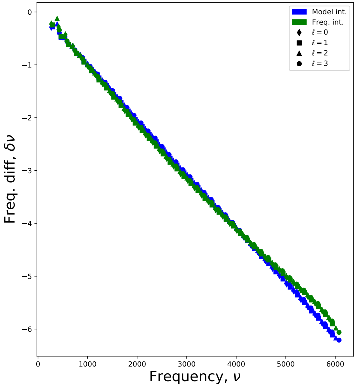
- non-dimensional echelle diagram of original models

Interpolation between tracks



- non-dimensional echelle diagram of interpolated models
- original radius: 6.7984×10^{10} cm
- interpolated radius (via mean density): 6.8032×10^{10} cm
- interpolated radius (simple): 6.8046×10^{10} cm

Interpolation between tracks



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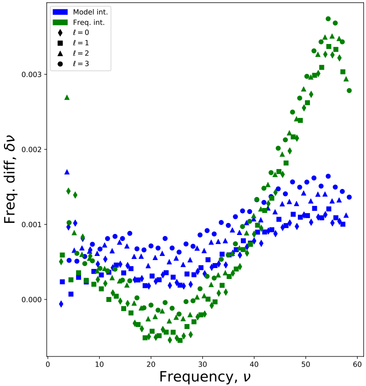

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Obtaining interpolation coefficients

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Obtaining interpolation coefficients

- is it better to separate age interpolation from interpolating other variables?
- in what follows, we will assume the two are separate

Interpolation between evolutionary tracks

Two options

- multilinear interpolation on a Cartesian grid
- triangulation

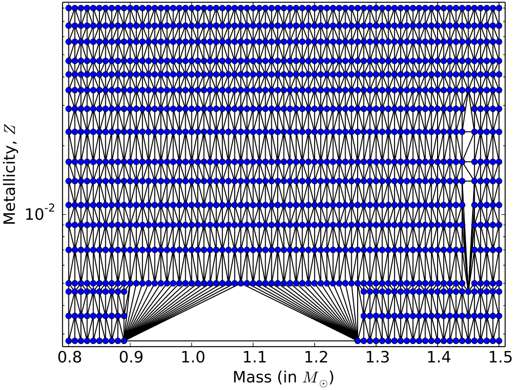
Interpolation between evolutionary tracks

Two options

- multilinear interpolation on a Cartesian grid
  - potentially more accurate
  - more restrictive on the choice of grid
  - combining 2^d models
- triangulation
  - potentially less accurate
  - more flexible on the choice of grid
  - combining $d+1$ models

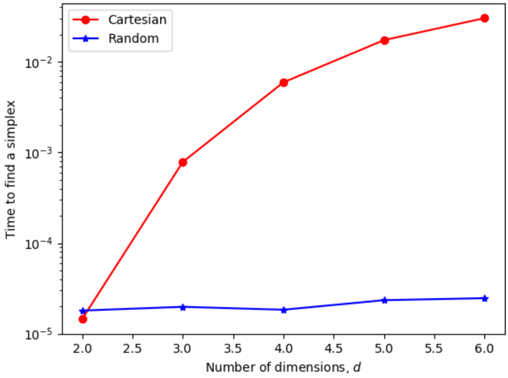
Interpolation using triangulation

- interpolation based on triangulation between different tracks



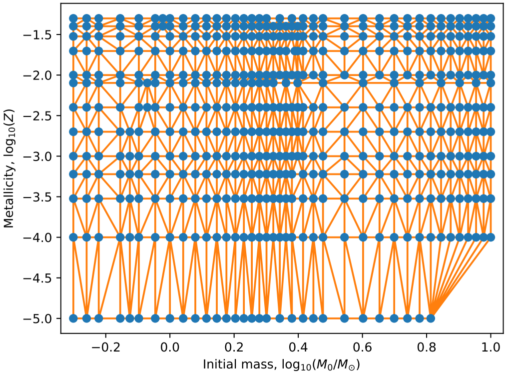
Interpolation using triangulation

- time for finding a simplex (i.e. a triangle, a tetrahedron etc.)



Interpolation using triangulation

- the difficulties and possible fixes for a Cartesian grid



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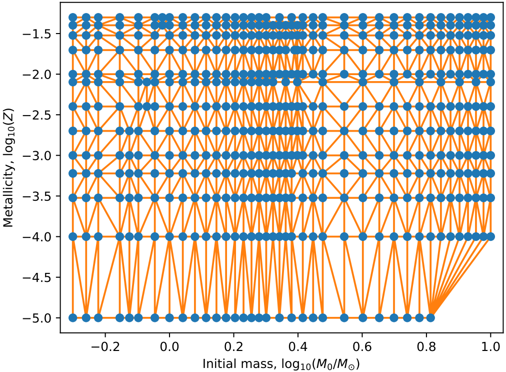

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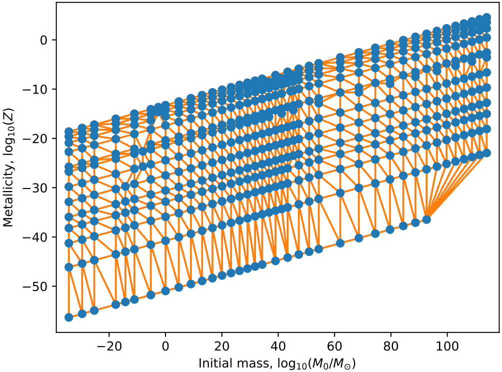

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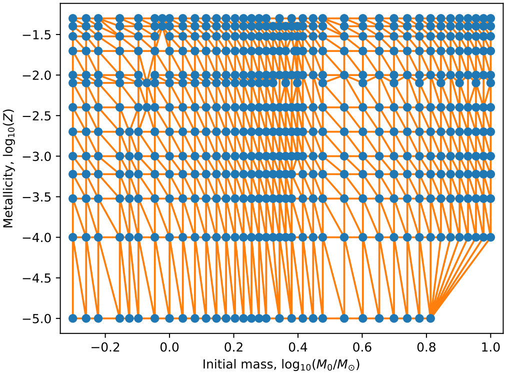

Interpolation using triangulation

Dear Daniel,

My guess is that find_simplex uses a gradient search and qhull's 'Qt' option to triangulate the output. Since your data contains mostly non-simplicial regions (i.e., more than d sites on a circumsphere of the region), the triangulation generates many coplanar regions for each non-simplicial region. The find_simplex code probably does not consider this case, and may visit all of the triangles for one non-simplicial region before finding an adjacent non-simplicial region to visit next.

Since the size of the Delaunay triangulation increases rapidly with dimension, this extra work likewise increases rapidly as the dimension increases.

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- any help in deciphering this answer?

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Interpolation along evolutionary tracks

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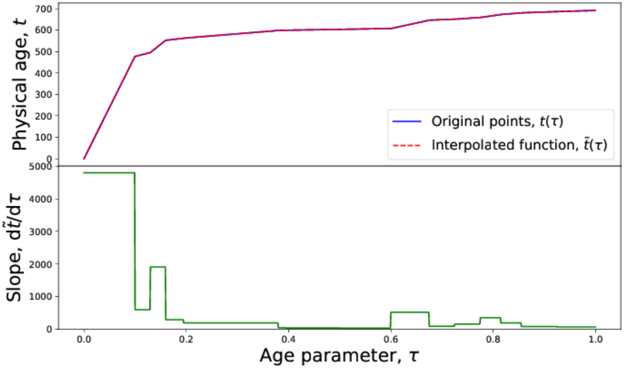
- **Question**: to what age should we interpolate?
- **Context**: age interpolation takes place before track interpolation
  - interpolate all tracks to same physical age
  - interpolate tracks to same "equivalent" age
- how does one define an equivalent age?
- beware of age prior

Equivalent age

- example of an equivalent age from BASTI



Weighting

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- how should one weight the classical and seismic constraints?
  - give every constraint the same weight
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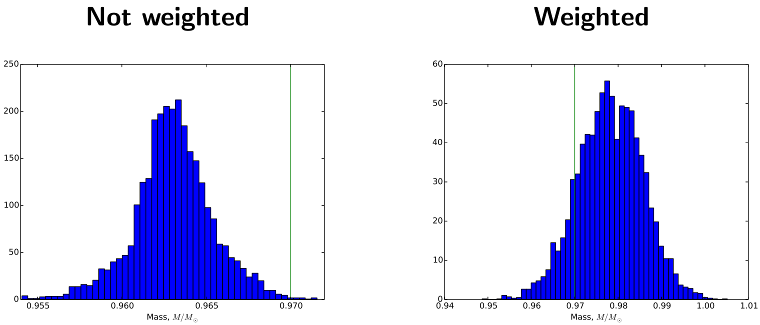
- **underlying problem**: our models are not accurate enough to reproduce the observed frequencies within their error bars

Weighting



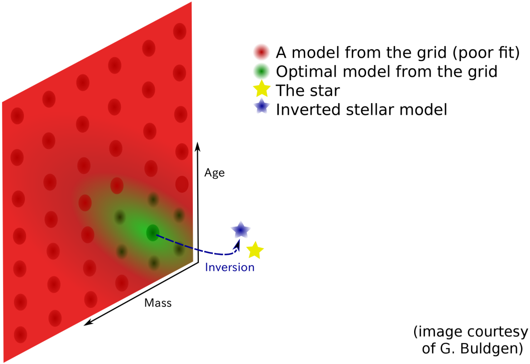
- weighting seems to produce more realistic error bars
- not weighting agrees better with observational error propagation (Rendle et al. in prep)

Error bars

- **Question**: which is better?
  - mass of the sun $= 0.995 \pm 0.0001$ $M_\odot$
  - mass of the sun $= 0.96 \pm 0.08$ $M_\odot$
- this question is also important when carrying out hare-and-hounds exercises and comparing different methods

Inverse analysis

- model-dependence of forward modelling
- inversions: one way to overcome this limitation



(image courtesy of G. Buldgen)

Inverse analysis – the stellar case

- inverse analysis has been extremely successful for the Sun
- much smaller number of pulsation modes available in other stars
  - may constrain rotation profile, but cannot carry out detailed structural inversions
  - invert for global quantities instead
- need reference models from forward analysis. How many?

Inversions of other integrated quantities

- these can reduce uncertainties on mass, radius, and age (Buldgen et al. 2016)

Quantity	**Purpose**	**Reference**
Mean density	help determine mass	Reese et al. (2012)
Acoustic radius	may help with glitches	Buldgen et al. (2015)
t indicator	evolutionary phase	Buldgen et al. (2015)
t_u indicator	evolutionary phase	Buldgen et al. (2015b)
$s_{\rm core}$ indicator	evol. phase & conv. cores	Buldgen et al. (2018)
$s_{\rm env}$ indicator	T and χ gradient at BCZ	Buldgen et al. (2018)

Glitch analysis

- may also provide less model-dependant results
- difficulties with aliasing – use forward analysis to decide between multiple solutions?
- He abundance from model calibration?

Conclusion

- different tools exist for carrying out forward analysis, inversions and glitch analysis
- more work is needed to combine and streamline these components
- the errors need to be quantified at every stage

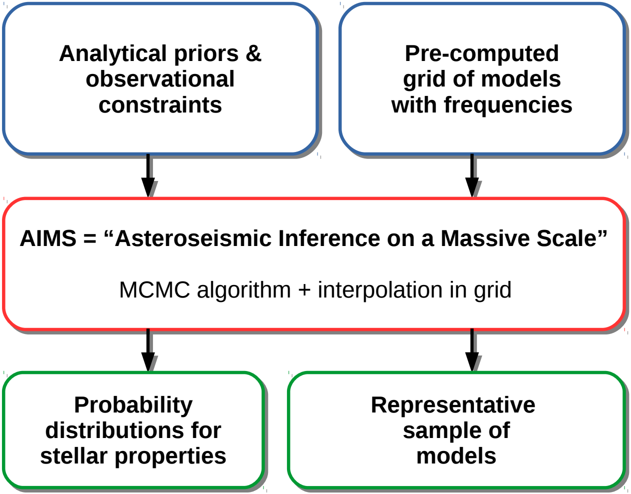
Forward analysis with AIMS

AIMS

- AIMS = "Asteroseismic Inferences on a Massive Scale"
- developed during my postdoc at the University of Birmingham with the help of colleagues

(logo by M. N. Lund)

Forward analysis with AIMS

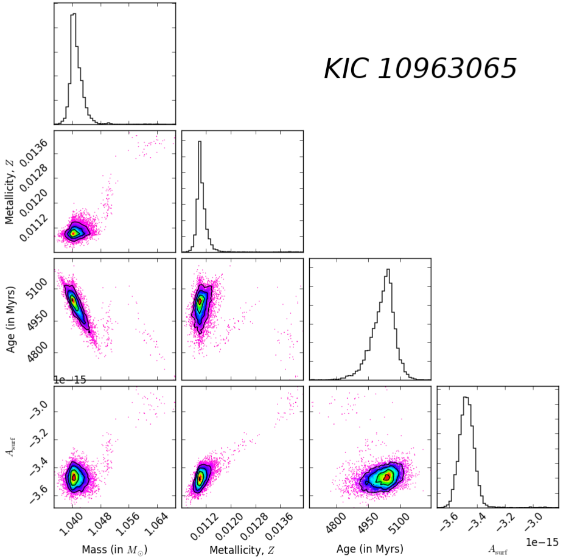


Forward analysis with AIMS

Specificity of AIMS – model interpolation

- advantages
  - not very time consuming (compared to calculating models on the fly)
  - partially overcomes the limitations of a finite grid resolution
- disadvantages
  - interpolation errors (which need to be quantified)

MCMC analysis



Various uses of AIMS

- applied to the Kepler LEGACY sample (Silva Aguirre et al. 2017)
- used in study of systematic effects and surface correction recipes (Nsamba et al. 2018)
- ongoing efforts to analyse interpolation errors (Rendle et al. in prep.)

Mean density inversions

- the difference in mass between the star and the reference model is:

$$\delta M = \int_0^R 4\pi \delta\rho r^2 \mathrm{d}r = \int_0^R 4\pi \rho r^2 \frac{\delta\rho}{\rho} \mathrm{d}r$$

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- the difference in mean density is:

$$\frac{\delta\bar{\rho}}{\bar{\rho}} = \int_0^1 4\pi x^2 \frac{R^3 \rho}{M} \frac{\delta\rho}{\rho} \mathrm{d}x,$$

where $x = r/R$ and $\bar{\rho} = 3M/(4\pi R^3)$ is the mean density

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  where $x = r/R$ and $\bar{\rho} = 3M/(4\pi R^3)$ is the mean density
- this last equation still applies even if the star and the model do not have the same radii

Mean density inversions

- structural modifications lead to frequency differences (derived from variational principle):

$$\frac{\delta\omega_{n,\ell}}{\omega_{n,\ell}} = \int_0^1 K^{n,\ell}_{\rho,\Gamma_1}(x)\frac{\delta\rho}{\rho}\mathrm{d}x + \int_0^1 K^{n,\ell}_{\Gamma_1,\rho}(x)\frac{\delta\Gamma_1}{\Gamma_1}\mathrm{d}x$$

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- a linear combination of the above equation for the different modes leads to:

$$\underbrace{\sum_i c_i \frac{\delta\omega_i}{\omega_i}} = \int_0^1 \underbrace{\left(\sum_i c_i K^i_{\rho,\Gamma_1}(x)\right)}_{K_{\mathrm{avg}}(x)} \frac{\delta\rho}{\rho}\mathrm{d}x + \int_0^1 \underbrace{\left(\sum_i c_i K^i_{\Gamma_1,\rho}(x)\right)}_{K_{\mathrm{cross}}(x)} \frac{\delta\Gamma_1}{\Gamma_1}\mathrm{d}x$$

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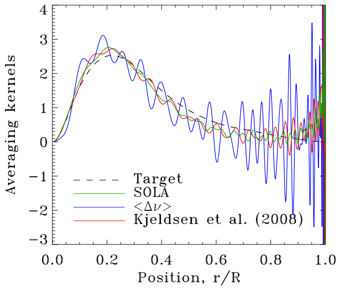
  where i is shorthand for $(n,\ \ell)$ and c_i are coefficients

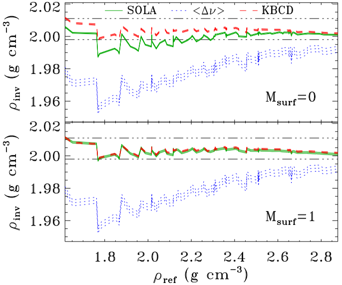
- use SOLA-type inversion (Pijpers & Thompson, 1992, 1994) to adjust c_i so as to reproduce $\delta\bar{\rho}/\bar{\rho}$

Mean density inversions

Averaging kernels



Inversion results



(Reese et al. 2012)