An improved calibration for the mixing length theory of convection

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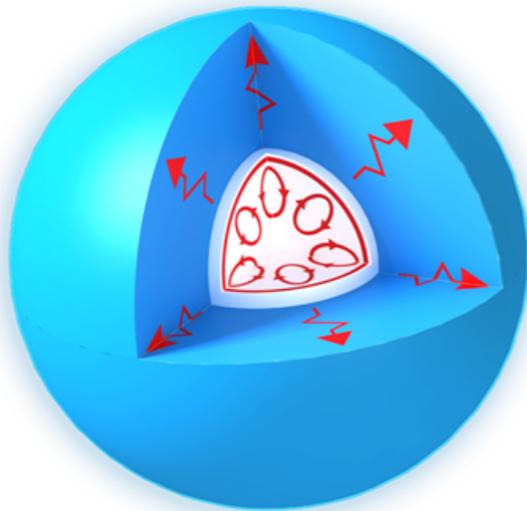

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In collaboration with: P. Demarque, J.D. Tanner, S. Basu (Yale)
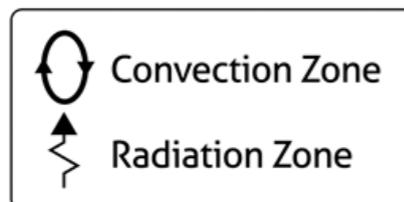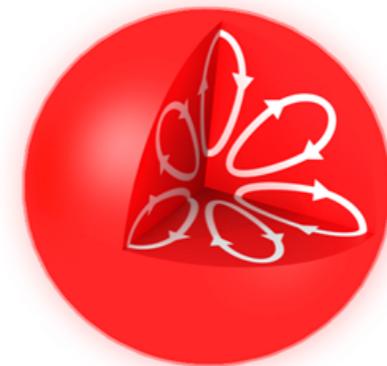
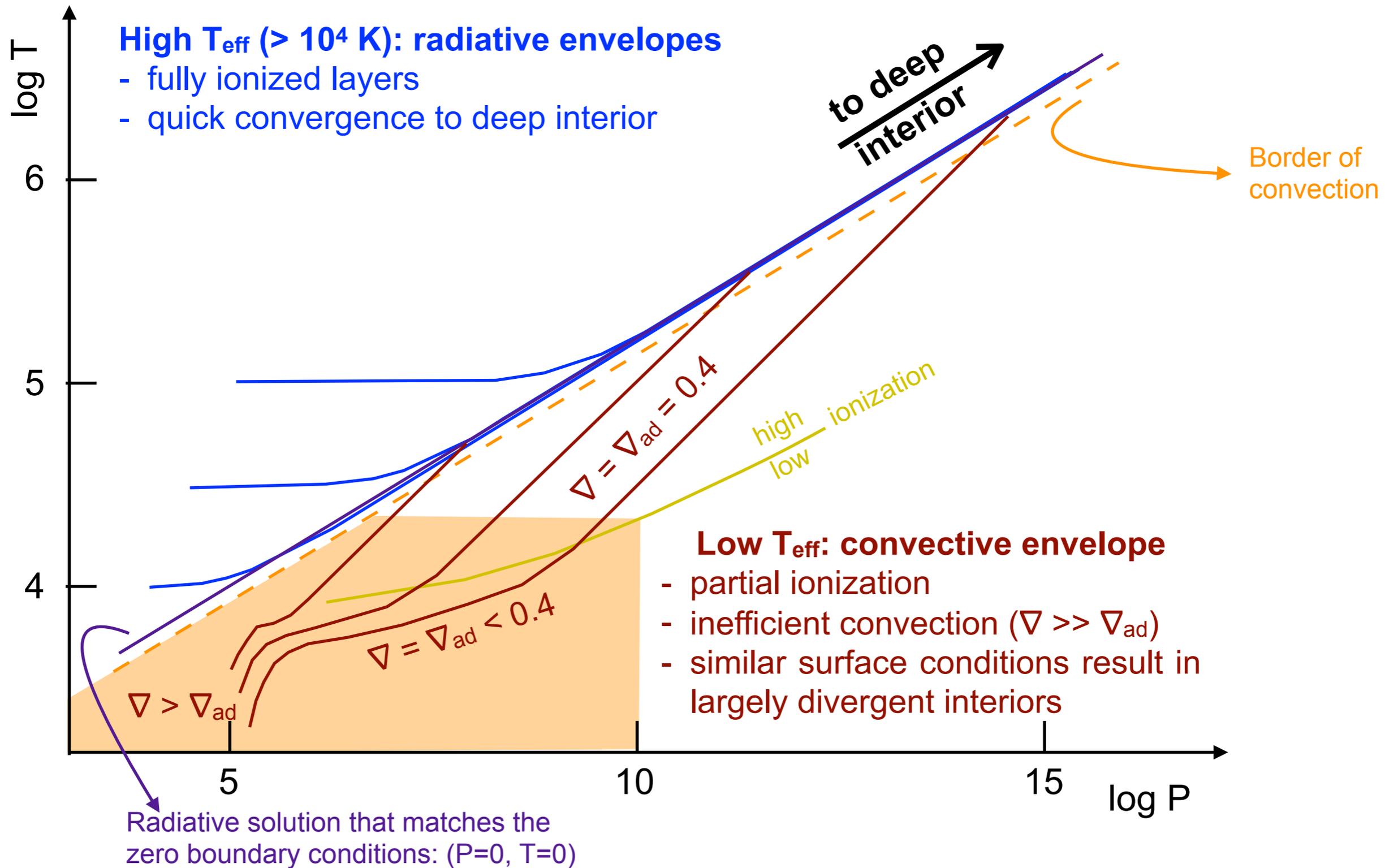# Convection in stars

> 1.5 solar masses

0.5 - 1.5 solar masses

< 0.5 solar masses

Convection Zone

Radiation Zone

Low-mass stars have convectively unstable outer layers

Outer CZs affect overall structure

High T$_{eff}$ (> 10^4 K): radiative envelopes
- fully ionized layers
- quick convergence to deep interior

to deep interior

Border of convection

$\nabla = \nabla_{ad} = 0.4$

high ionization
low ionization

Low T$_{eff}$: convective envelope
- partial ionization
- inefficient convection ($\nabla \gg \nabla_{ad}$)
- similar surface conditions result in largely divergent interiors

$\nabla = \nabla_{ad} < 0.4$

$\nabla > \nabla_{ad}$

log T

log P

Radiative solution that matches the zero boundary conditions: (P=0, T=0)
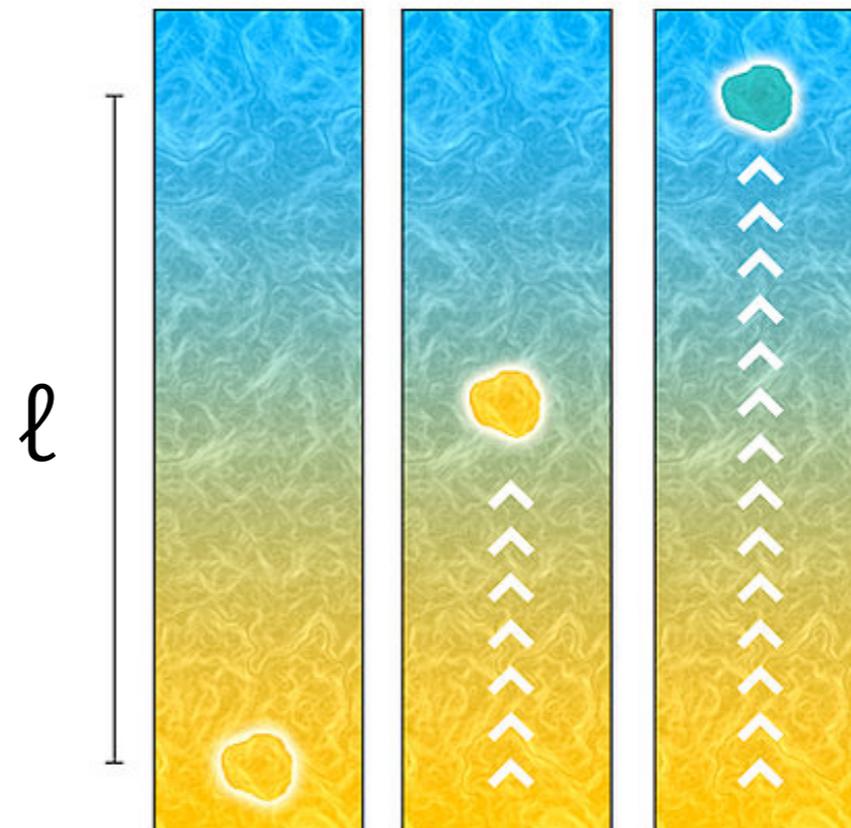
adapted from Kippenhahn & Weigert (1990)

Mixing Length Theory (MLT)

- MLT contains free parameter α

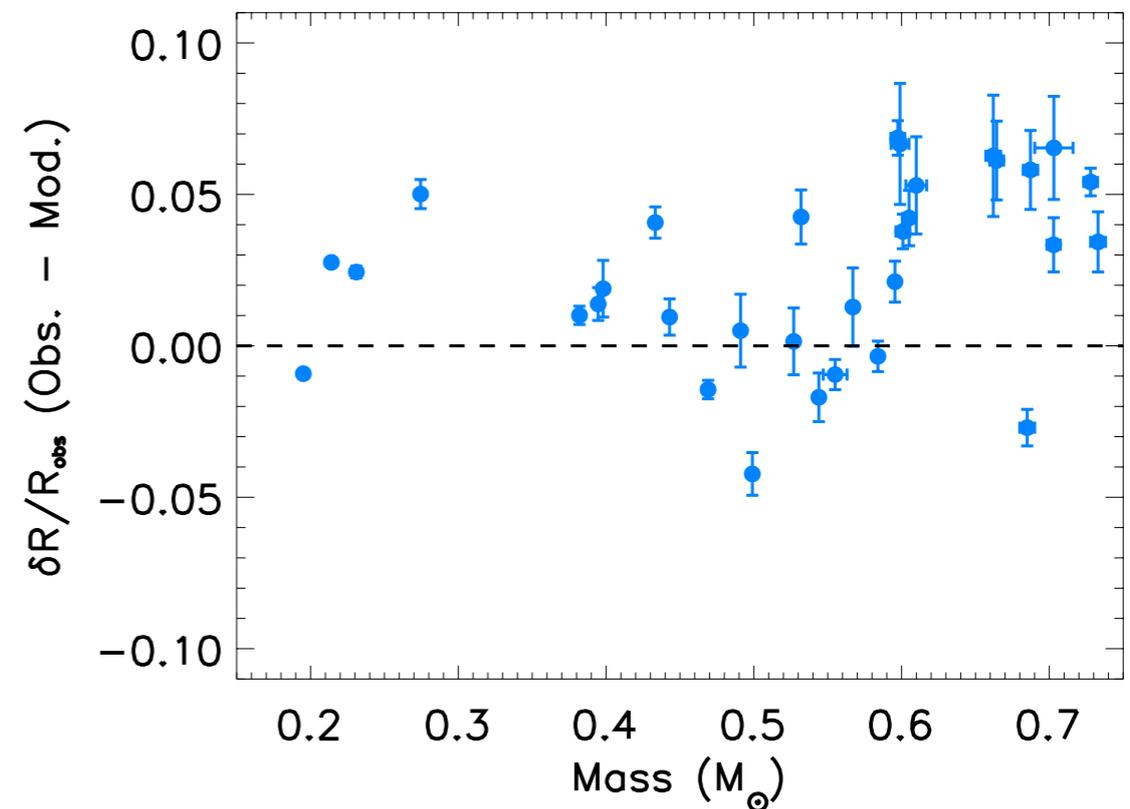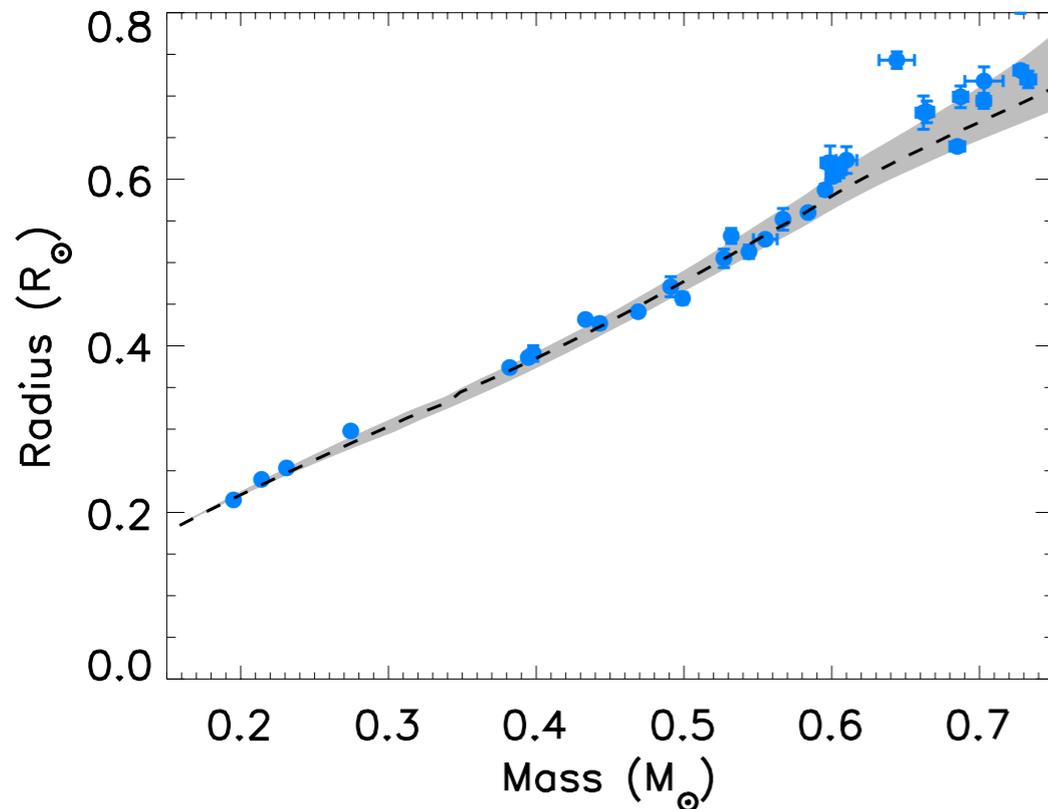- Standard calibration of α based on the Sun ($t_\odot$, $M_\odot$, $R_\odot$)

- Solar-calibrated α:

  - constant along evolutionary track

  - independent of other stellar parameters (e.g., mass, evolutionary phase, metallicity)

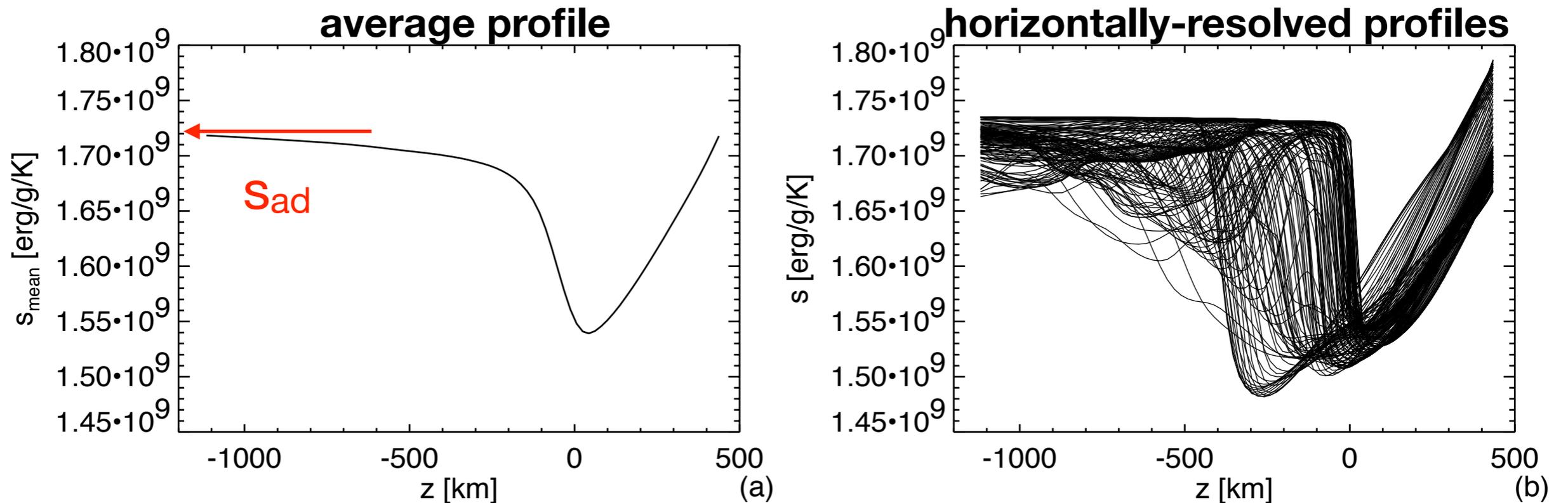**very strong assumptions!**

$$\alpha = \ell/H_P$$

Radius inflation of low-mass stars

- M < 0.7 M⊙ stars have radii larger than model predictions by ~ 3-5%

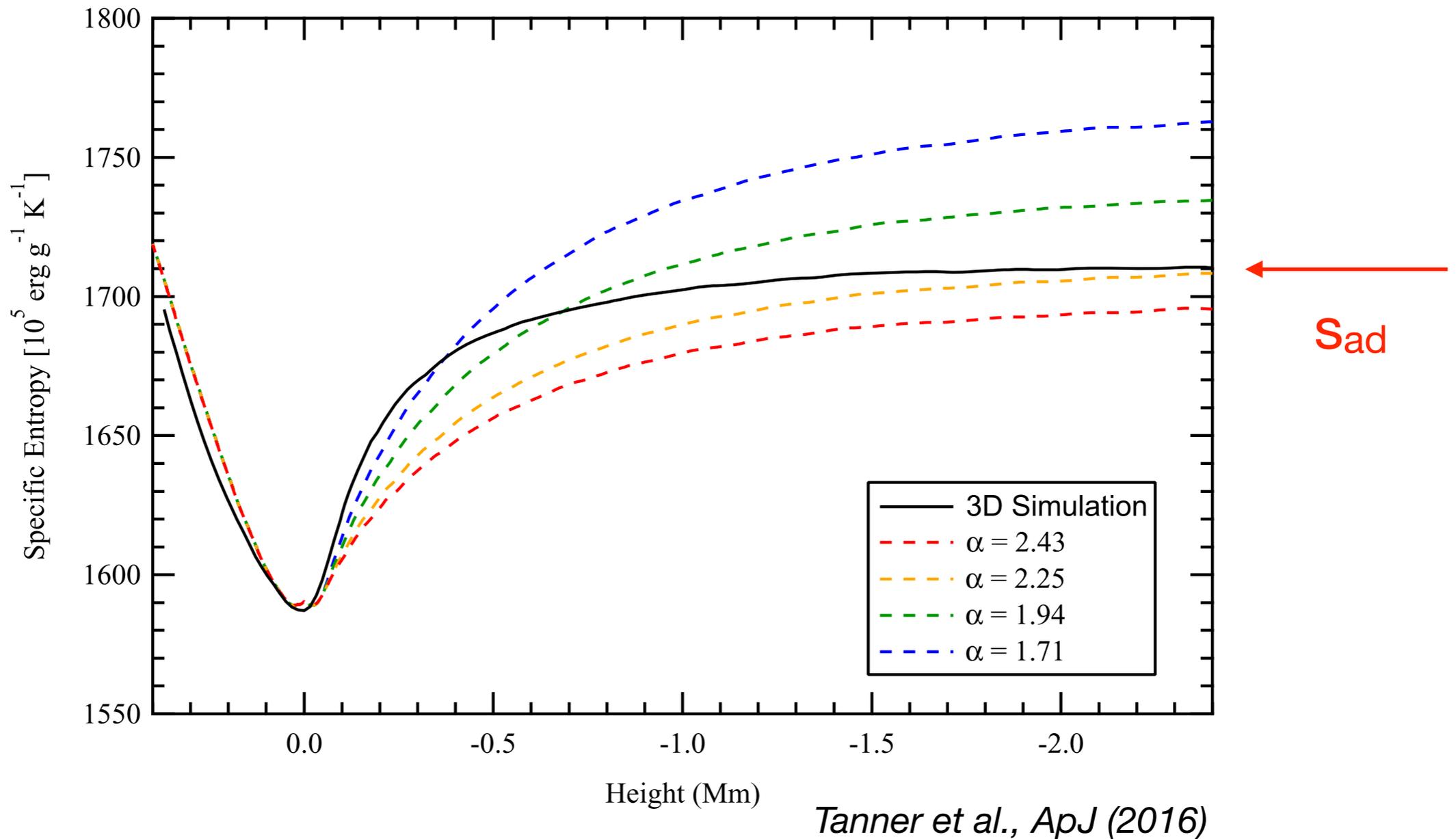- Rotational/magnetic effects on convection are a natural explanation

adapted from Spada et al., ApJ (2017)

3D RHD simulations of CZs

average profile

horizontally-resolved profiles

Ludwig et al., A&A (1999)

s_{ad}: adiabatic specific entropy (units: erg g^{-1} K^{-1}) in the deep convective layers

Calibration of α using RHD simulations

Tanner et al., ApJ (2016)

Tune α to reproduce the value of s_{ad} obtained from RHD simulations

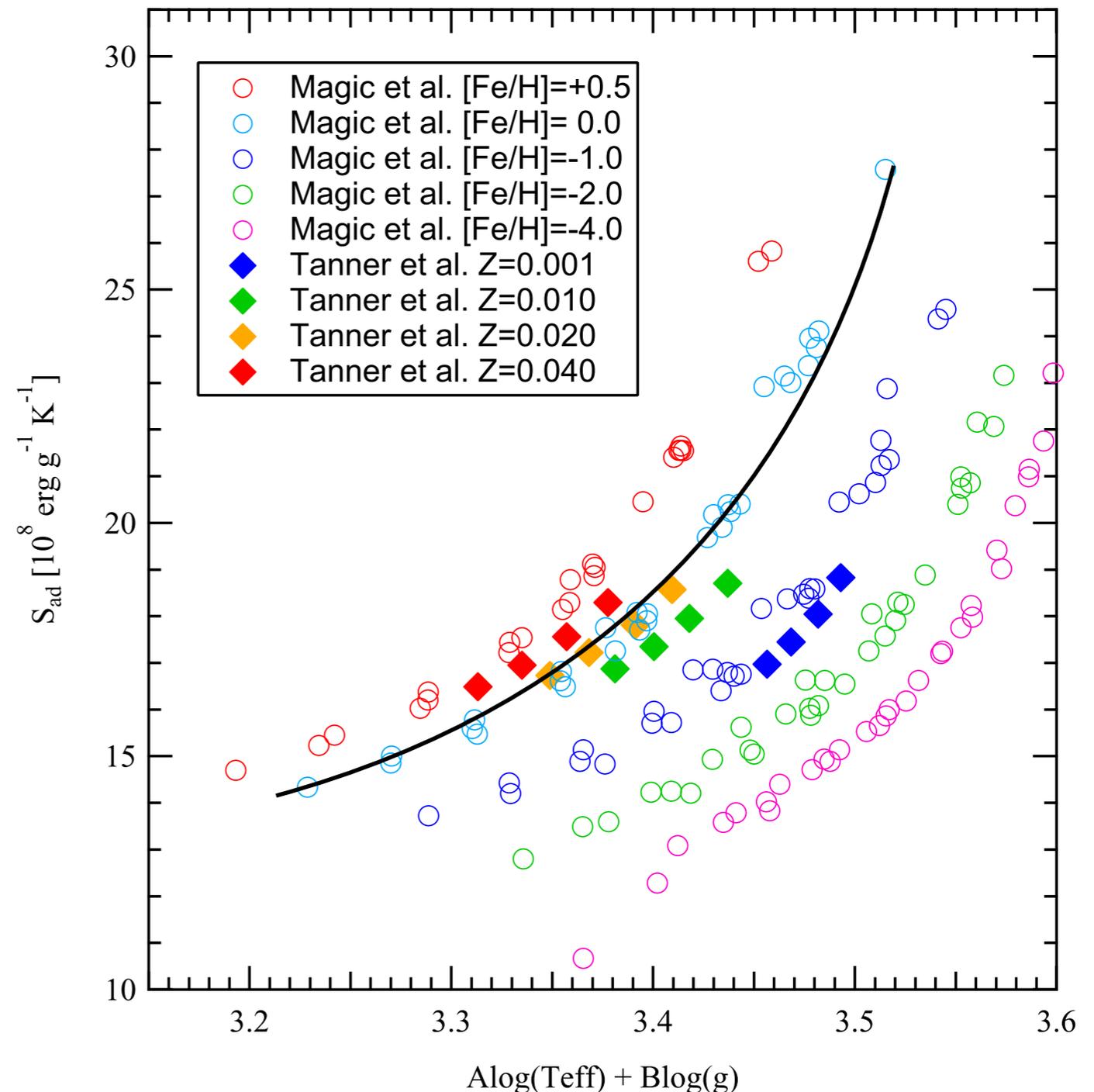# Extract s$_{\text{ad}}$ from 3D RHD simulations

One-dimensional fit:

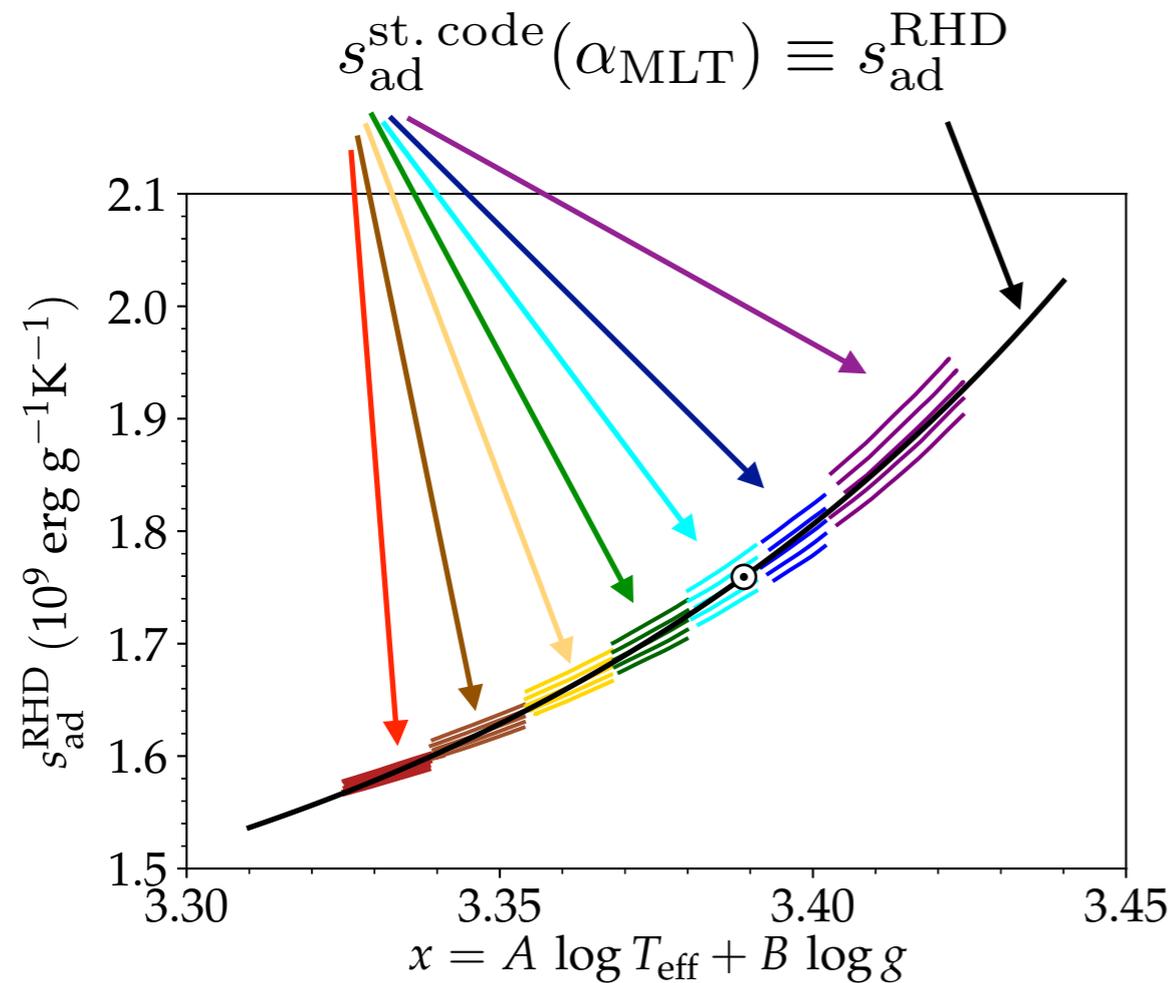$$s_{\text{ad}}^{\text{RHD}} = s_0 + \beta \exp\left(\frac{x - x_0}{\tau}\right)$$

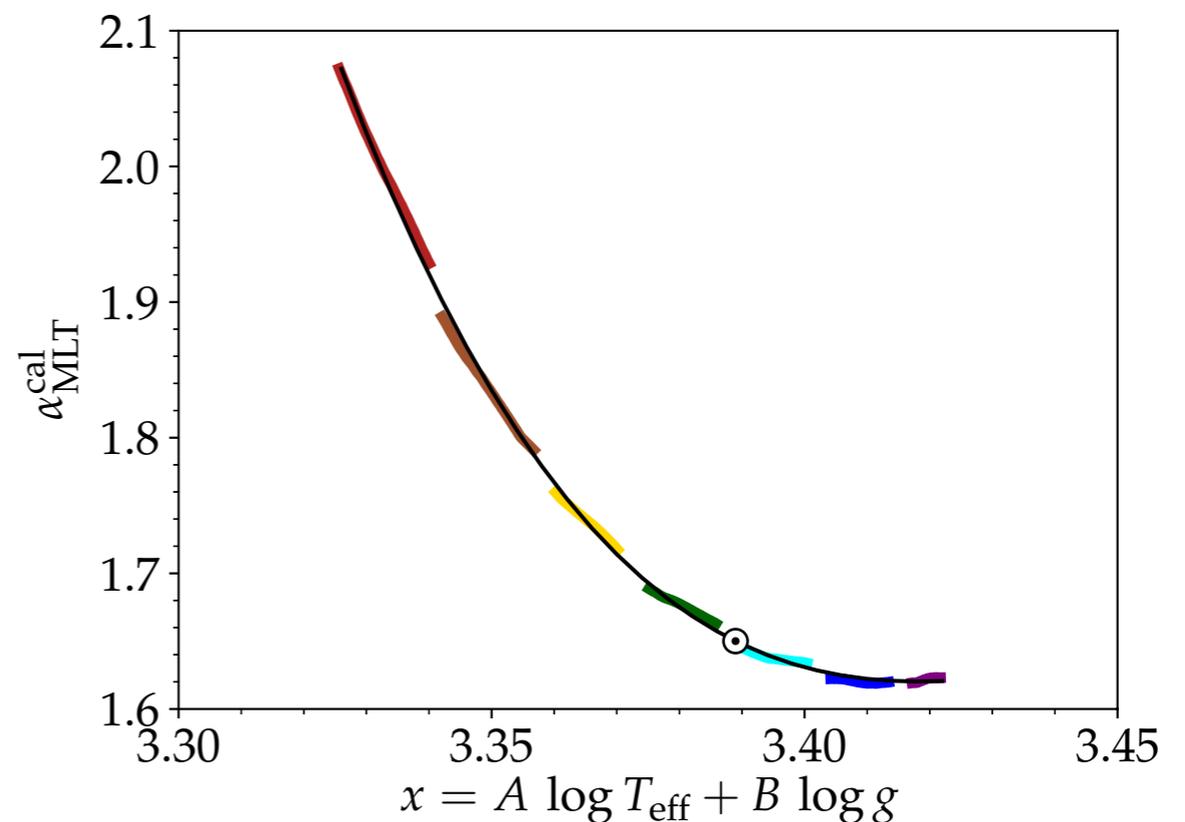as a function of the variable:

$$x = A \log T_{\text{eff}} + B \log g$$

The parameters of the fit
are metallicity-dependent

Legend:
- Magic et al. [Fe/H]=+0.5
- Magic et al. [Fe/H]= 0.0
- Magic et al. [Fe/H]=-1.0
- Magic et al. [Fe/H]=-2.0
- Magic et al. [Fe/H]=-4.0
- Tanner et al. Z=0.001
- Tanner et al. Z=0.010
- Tanner et al. Z=0.020
- Tanner et al. Z=0.040

Axis labels: S_{ad} [10^8 erg g^{-1} K^{-1}] ; Alog(Teff) + Blog(g)

Tanner et al., ApJ (2016)

Entropy-based calibration of α

$$s_{\mathrm{ad}}^{\mathrm{st.\,code}}(\alpha_{\mathrm{MLT}}) \equiv s_{\mathrm{ad}}^{\mathrm{RHD}}$$

$$\to \alpha_{\mathrm{MLT}}^{\mathrm{cal}}([\mathrm{Fe/H}], x).$$

Spada, Demarque, Basu, and Tanner (in preparation)

Implementation in a stellar code (YREC)

Construction of an entropy-calibrated evolutionary track:
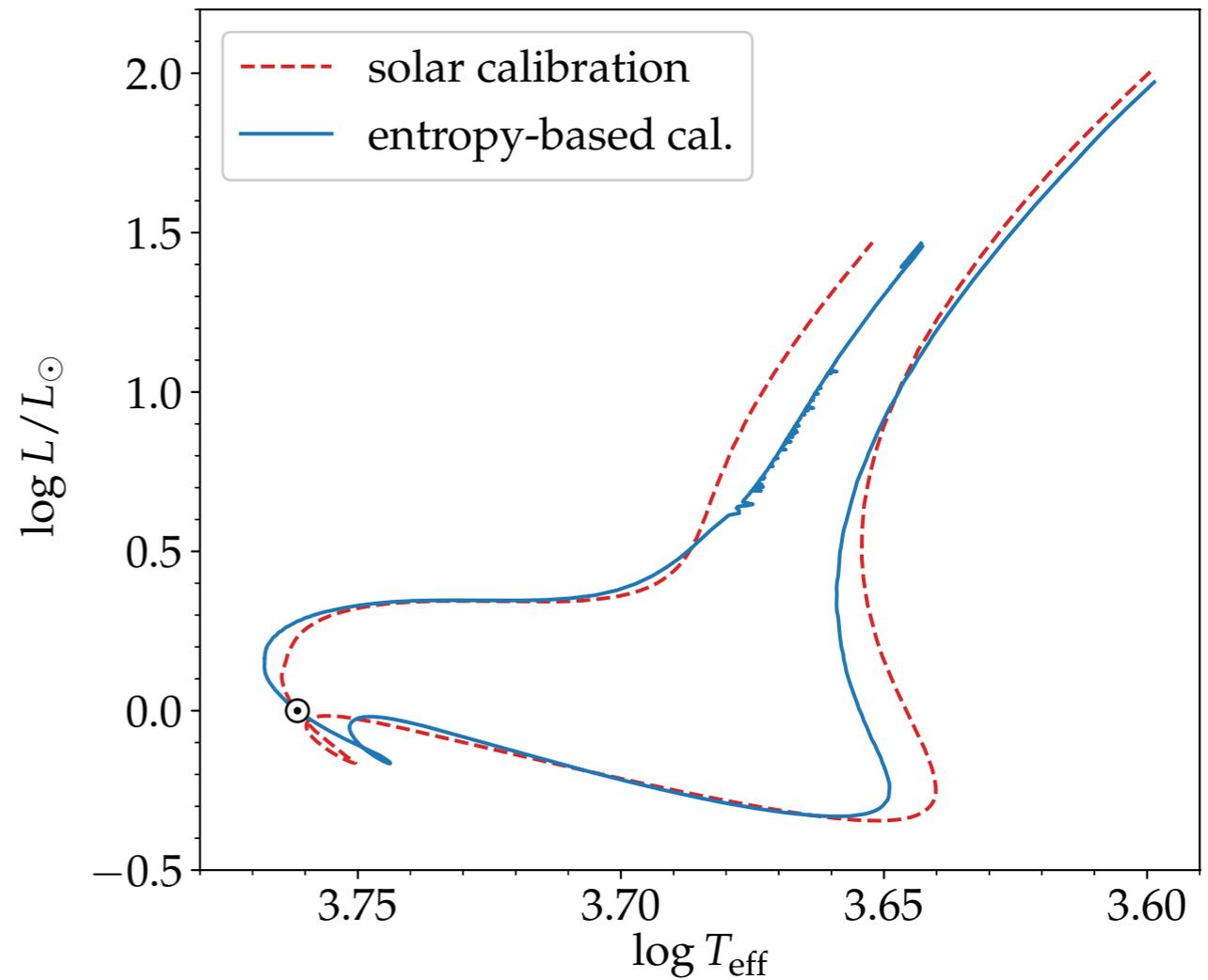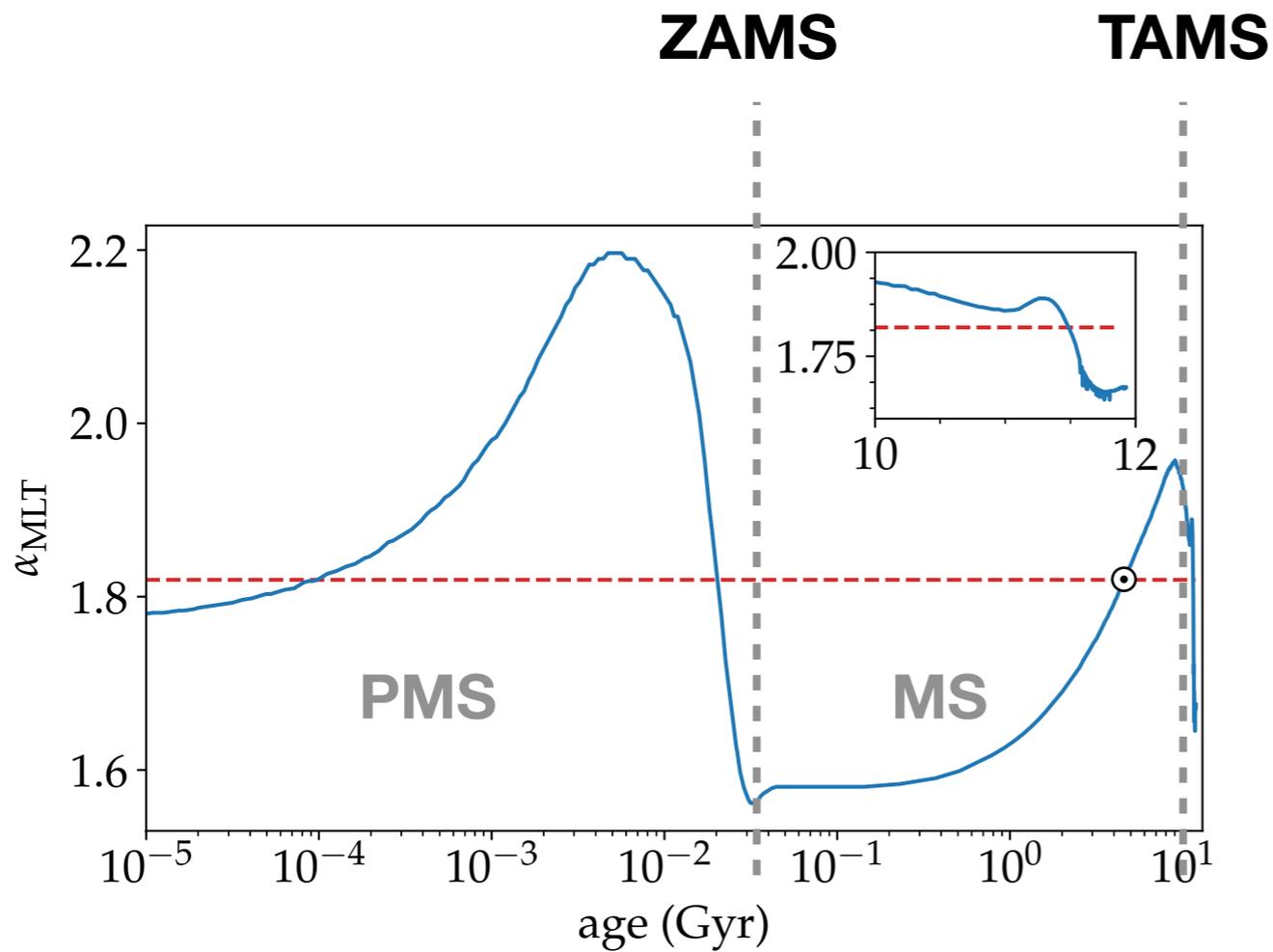
At each evolutionary step:

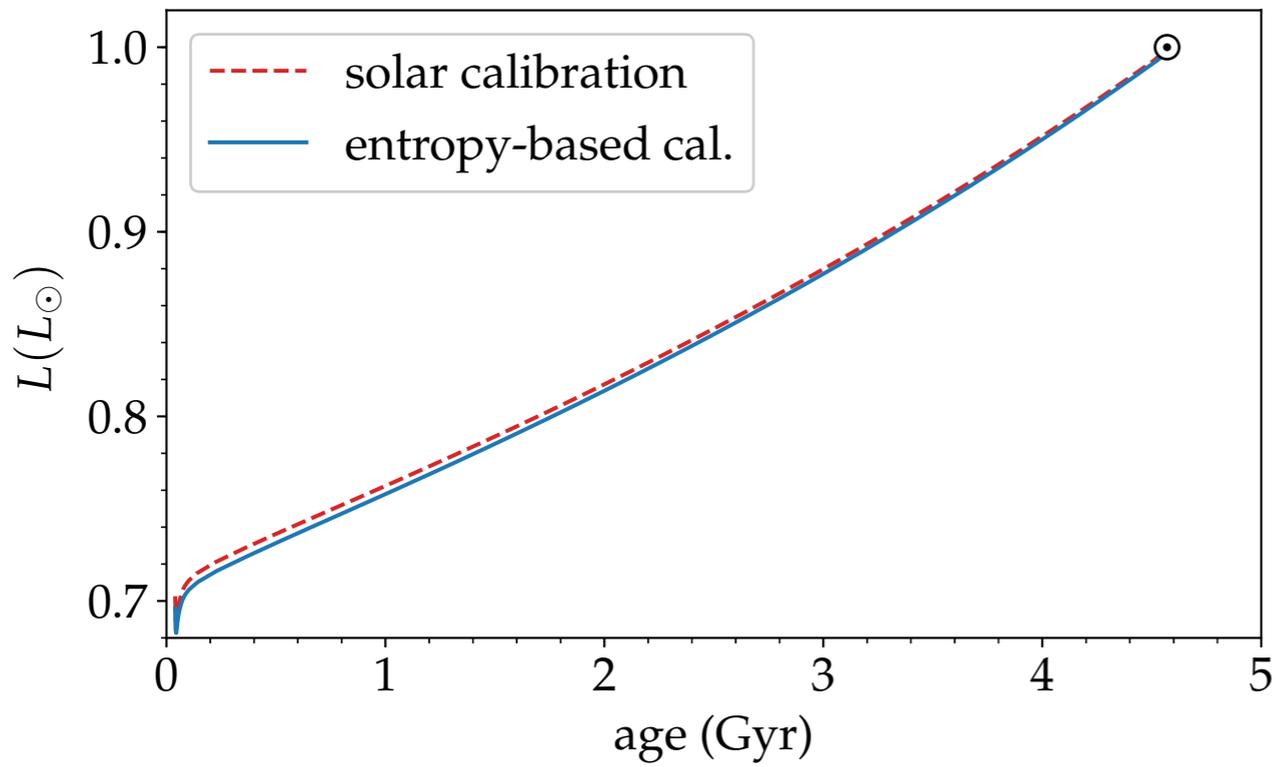- From the current stellar parameters, evaluate:

$$s_{\mathrm{ad}}^{\mathrm{RHD}} = s_0 + \beta \exp\left(\frac{x - x_0}{\tau}\right)$$
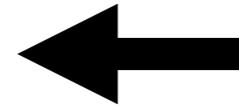
- Iteratively solve (e.g., bisection method):

$$s_{\mathrm{ad}}^{\mathrm{st.\,code}}(\alpha_{\mathrm{MLT}}) \equiv s_{\mathrm{ad}}^{\mathrm{RHD}} \rightarrow \alpha_{\mathrm{MLT}}^{\mathrm{cal}}([\mathrm{Fe/H}], x)$$

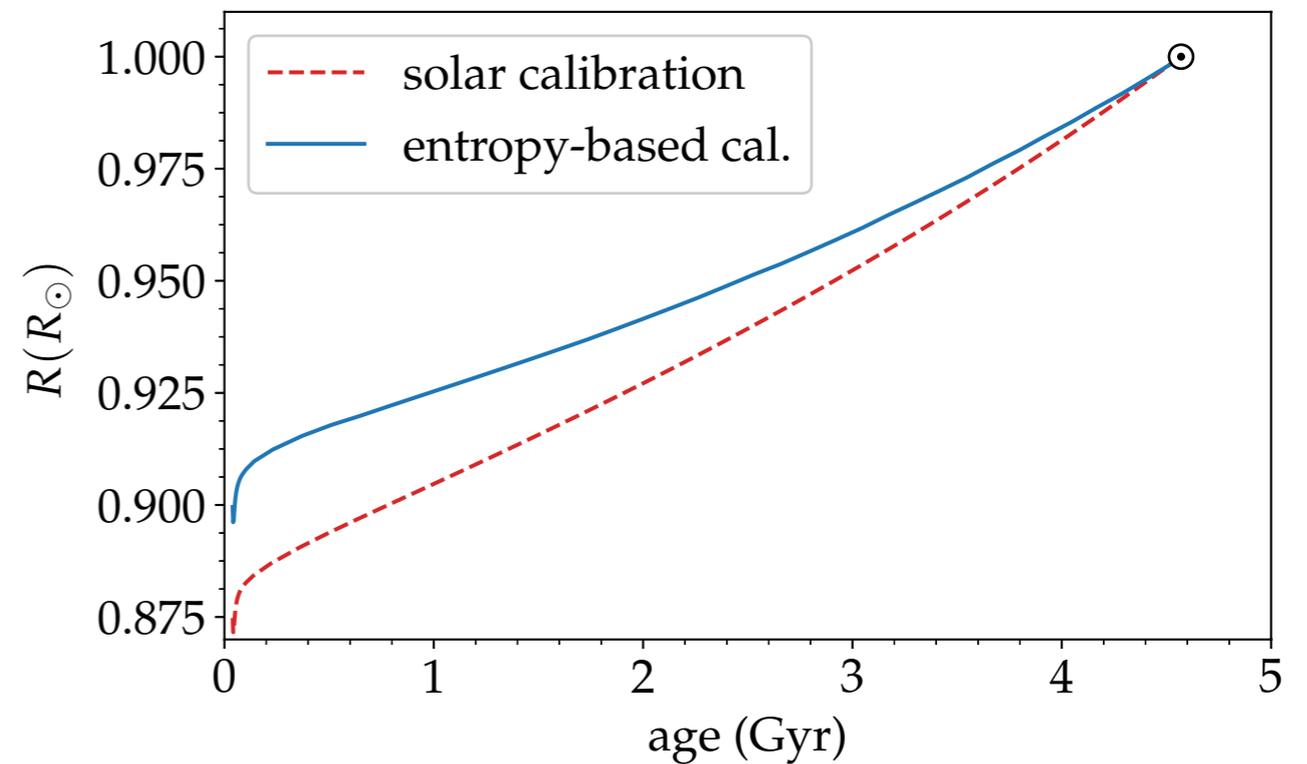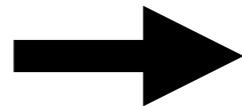# 1 M$_\odot$ evolutionary track

Spada, Demarque, Basu, and Tanner (in preparation)

Luminosity is unaffected

Significant impact on radius, properties of the CZ (e.g., depth, bottom temperature, s_{ad})

Spada, Demarque, Basu, and Tanner (in preparation)

Impact on light elements depletion

Li destruction during the PMS is affected (~factor of 2)
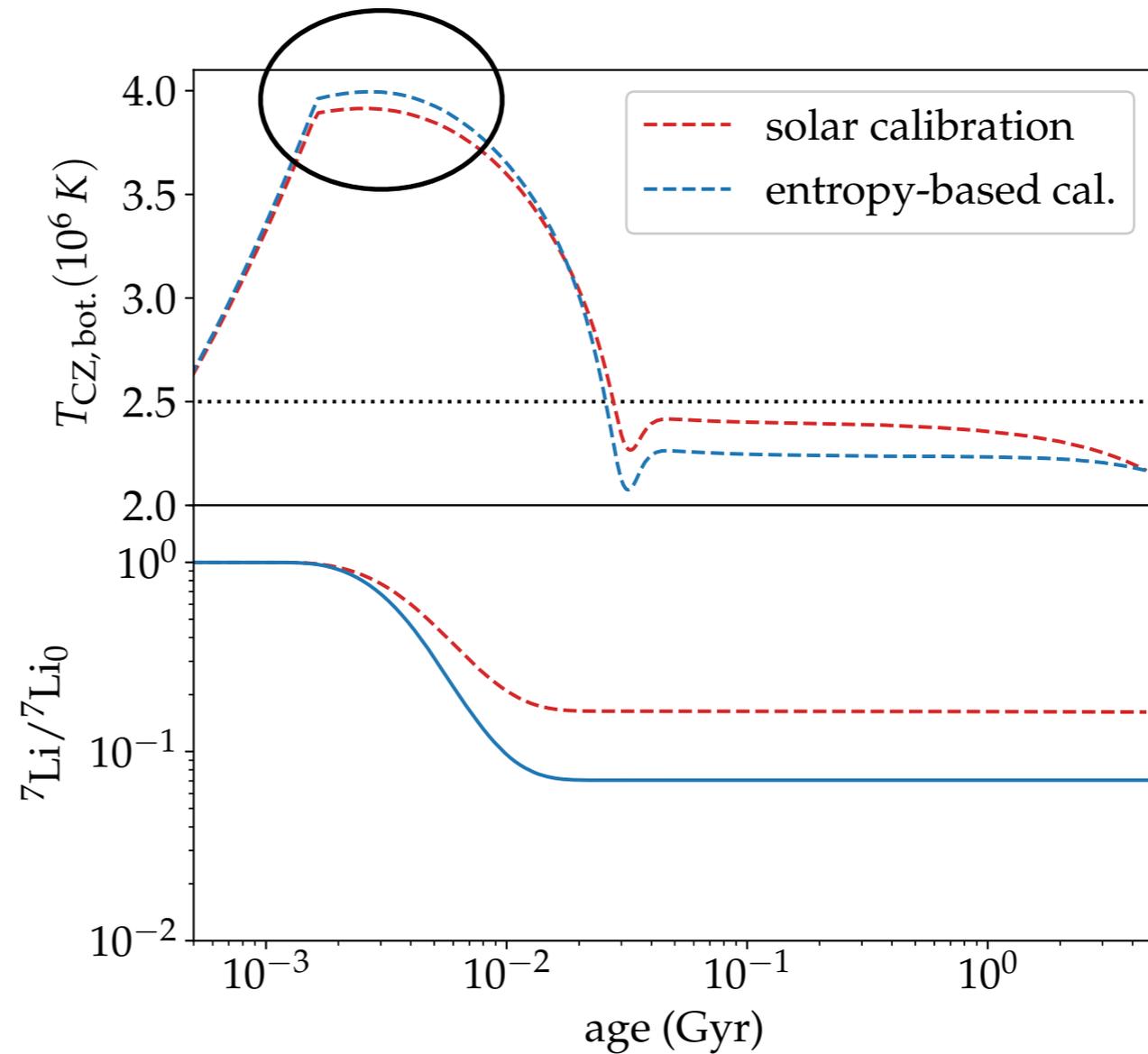
Final Remarks

- Entropy-based α yields an improved calibration for the radii of low-mass stars (radius discrepancy problem)

- More physically-motivated than solar-calibration: comparing α across stellar evolution codes is meaningless

- Naturally takes into account effect on convection of stellar parameters (mass, surface composition, evolutionary phase)

- Yields improved evolutionary history of the Sun (potential impact on dynamics, habitability of solar system objects)